Connection between decoherence and fidelity decay in echo dynamics

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Entanglement between a quantum system and its environment leads to loss of coherence in the former. In general, the temporal fate of coherences is complicated. Here, we establish the connection between decoherence of a central system and fidelity decay in the environment for a variety of situations, including both energy conserving and dissipative couplings. We show how properties of unitary time evolution of the environment can be inferred from the nonunitary evolution of coherences in the central system. This opens up promising ways for measuring Loschmidt echoes in a variety of situations.

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I. INTRODUCTION

Quantum Loschmidt echoes [1] have received a large amount of theoretical and experimental attention in recent years [2–5]. Such echoes are obtained by propagating an initial state for some time t forwards and then backwards in time. In the ideal situation where the forward and backward evolutions are the same, the system ends up in its initial state at time $2t$. However, in reality the forward and backward evolution are distorted by inherently uncontrollable perturbations. These deviations typically add up in the course of the evolution, which results in a final state being notably different from the initial state. A natural measure for this difference is the overlap of both states, the *fidelity amplitude*. Its absolute value squared is the *fidelity*. Experimentally, this concept of Loschmidt echoes has been widely used in connection with nuclear magnetic resonance spectroscopy, photon echoes and wave packet echoes of trapped atoms [5].

Alternatively, fidelity can be looked at from a different viewpoint. Namely, one may consider two identical initial states being propagated according to slightly different Hamiltonians. Then, after time t the overlap between the two states will no longer be equal to one. Formally this quantity is again a fidelity amplitude. In this picture the relation between the separation of nearby trajectories in classical dynamics (as a measure of chaos) and fidelity decrease becomes most transparent [2–5].

While fidelity is based on the unitary time evolution of the quantum system of interest, *decoherence* arises due to the coupling to additional "environmental" degrees of freedom, *i.e.* due to the growing entanglement between that system and its "environment." Decoherence as a dynamical phenomenon has received growing attention in the last few years [6–8]. The reason is obvious: For newly emerging quantum technologies, such as quantum cryptography and quantum computing, or quantum information processing in general, the stability of quantum coherence is fundamental [9]. Decoherence is *the* obstacle that has to be overcome for these technologies to prove successful. This requires a clear understanding of mechanisms and time scales involved.

In this work we investigate situations where decoherence in the central system can be related to fidelity decay in the environment. We shall show that this connection between apparently unrelated research areas is quite general. The principal idea, is to use an internal degree of freedom both to create the difference between the two Hamiltonians involved and to monitor the fidelity decay in the course of the evolution. An experimental configuration which allows to realize a fidelity measurement of this type has been proposed in [10,11]. We investigate various situations where it is possible to interpret coherences (off-diagonal elements of the reduced density operator) in one subsystem as fidelity amplitudes of unitary, perturbed dynamics in the other. The strength of the perturbation may be related to the "distance" of the initially superposed states, as will be explained below.

The argument is based on the unitary evolution in the product Hilbert space $\mathcal{H}=\mathcal{H}^{\mathrm{c}}\otimes\mathcal{H}^{\mathrm{e}}$, of the Hilbert spaces for the central system (c) and the environment (e), respectively. We consider a total Hamiltonian of the form

$$H = H^{\mathrm{c}} + H^{\mathrm{e}} + H^{\mathrm{int}} \tag{1}$$

consisting of two Hamiltonians that describe the two subsystems separately, and an interaction term H^{int}, for which we shall consider different forms, as specified below. Note that up to this point, the designations as "environment" and "central system" are purely conventional. The only important point is the existence of two spaces. Over one of these, i.e., the "environment," we shall execute partial traces to consider the entanglement between the two spaces in terms of the off-diagonal matrix elements of the density matrix in the other space, i.e., the "central system."

In Sec. II we consider a coupling H^{int} that conserves the energy of the central system. The environmental influence is thus not of the dissipative type; still, phase relations in the central system will be disturbed and thus coherences lost. This setting is a generalization of recent proposals and experimental realizations in the literature. In Sec. III we turn our attention to the damped harmonic oscillator, i.e., to the so-called amplitude coupling between a central oscillator and a "bath" of environmental oscillators. We show that a similar connection between decoherence and fidelity decay may hold even in this dissipative case, where the coupling (H^{int}) and

the Hamiltonian of the central system (H^c) do not commute. The famous Paris decoherence experiment [12] may thus be interpreted as an environmental "Loschmidt-echo" experiment. Finally, in Sec. IV we consider more general situations where the relation between decoherence and environmental echo is only approximately valid. This is the case, if decoherence alias fidelity decay is fast, compared to typical time scales in the isolated central system.

II. ENERGY CONSERVING COUPLING—"DEPHASING"

An energy conserving coupling for the central system is realized when the coupling term in Eq. (1) is of the form

$$H^{\rm int} = \sum_j |\phi_j\rangle\langle\phi_j| \otimes V_j^{\rm e}, \tag{2}$$

where the $\{\phi_j\}$ form a complete set of eigenstates of H^c (for convenience, we shall assume that the spectrum of H^c is discrete). In the eigenbasis representation, $H^c = \Sigma_j |\phi_j\rangle \varepsilon_j \langle\phi_j|$, the full Hamiltonian may be written as

$$\begin{aligned} H &= \sum_j |\phi_j\rangle\varepsilon_j\langle\phi_j| \otimes \mathbf{1} + \mathbf{1} \otimes H^{\rm e} + \sum_j |\phi_j\rangle\langle\phi_j| \otimes V_j^{\rm e} \\ &= \sum_j |\phi_j\rangle\langle\phi_j| \otimes [\varepsilon_j \mathbf{1} + H^{\rm e} + V_j^{\rm e}]. \end{aligned} \tag{3}$$

As the Hamiltonian H^c commutes with H, the energy of the central system is conserved. Hence, the eigenstates of H must be separable: $|\Psi\rangle = |\phi_j\rangle \otimes |\chi_j^\alpha\rangle$, where the wave functions $|\chi_j^\alpha\rangle$ satisfy the j-dependent Schrödinger equation

$$[H^{\rm e} + V_j^{\rm e} + \varepsilon_j]|\chi_j^\alpha\rangle = E_j^\alpha |\chi_j^\alpha\rangle \tag{4}$$

in the Hilbert space of the environment.

Time evolution and fidelity. Since H^c is a constant of motion, an initial product state $|\Psi_0\rangle = |\phi_j\rangle \otimes |\chi_0\rangle$ with an eigenfunction $|\phi_j\rangle$ of H^c will remain a product state for all times. We find

$$|\Psi(t)\rangle = {\rm e}^{-{\rm i}\varepsilon_j t}|\phi_j\rangle \otimes |\chi_j(t)\rangle \tag{5}$$

with the environmental state $|\chi_j(t)\rangle$ obeying the j-dependent Schrödinger equation

$$\mathrm{i}\hbar\partial_t|\chi_j(t)\rangle = [H^{\rm e} + V_j^{\rm e}]|\chi_j(t)\rangle \tag{6}$$

in the Hilbert space of the environment. Clearly, the initial state $|\chi_j(0)\rangle = |\chi_0\rangle$ is independent of j.

In general, an initially separable state $|\Psi_0\rangle = |\phi_0\rangle \otimes |\chi_0\rangle$ will not remain separable under time evolution. Using the eigenbasis of H^c we write $|\phi_0\rangle = \Sigma_j a_j |\phi_j\rangle$ and find from the previous considerations the entangled state

$$|\Psi(t)\rangle = \sum_j a_j {\rm e}^{-{\rm i}\varepsilon_j t}|\phi_j\rangle \otimes |\chi_j(t)\rangle. \tag{7}$$

Crucially, the "perturbing potential" $V_j^{\rm e}$ that governs the evolution of the environmental states depends on the choice of the H^c-eigenstate $|\phi_j\rangle$. We also see that $|\chi_j(t)\rangle$ evolves unitarily, so that its norm is conserved. From Eq. (7) we compute the reduced density matrix $\varrho^c(t)$ in the central system:

$$\begin{aligned} \varrho^{\rm c}(t) &= {\rm Tr}_{\rm e}|\Psi(t)\rangle\langle\Psi(t)| \\ &= {\rm Tr}_{\rm e}\sum_{jk} a_j a_k^* {\rm e}^{-{\rm i}(\varepsilon_j-\varepsilon_k)t}|\phi_j\rangle\langle\phi_k| \otimes |\chi_j(t)\rangle\langle\chi_k(t)|. \end{aligned} \tag{8}$$

Hence, coherences between eigenstates $|\phi_j\rangle$, $|\phi_k\rangle$ of the central system are given by the matrix elements:

$$\varrho_{jk}^{\rm c}(t) = {\rm e}^{-{\rm i}(\varepsilon_j-\varepsilon_k)t}\langle\chi_k(t)|\chi_j(t)\rangle\varrho_{jk}^{\rm c}(0). \tag{9}$$

The decay of coherences is thus determined by the decay of the fidelity amplitude in the Hilbert space of the environment, for which we can write

$$\langle\chi_k(t)|\chi_j(t)\rangle = \langle\chi_0|M(t)|\chi_0\rangle, \tag{10}$$

where $M(t) = \tilde{U}_0(-t)\tilde{U}(t)$ is a so-called echo operator [3,13], while $\tilde{U}_0(t)$ and $\tilde{U}(t)$ are the respective evolution operators for the Hamiltonians:

$$\tilde{H}_0 = H^{\rm e} + V_k^{\rm e}, \;\; \tilde{H} = H_0 + V_j^{\rm e} - V_k^{\rm e}. \tag{11}$$

Note that each nondiagonal matrix element of ϱ^c involves a different echo operator with (slightly) different $\tilde{H}_0$ and $\tilde{H}$. However, in many cases the initial coefficients a_j as well as the phases $\exp[-{\rm i}(\varepsilon_j - \varepsilon_k)t]$ can be controlled quite precisely (see subsection of Sec. II), so that coherences and the corresponding fidelities are readily identified in actual experiments.

We may finally mention a special situation of interest, where the environment factor of the separable interaction is simply proportional to the Hamiltonian of the environment, *i.e.* $V_j^{\rm e} = f_j H^{\rm e}$ with some real number f_j. In this case Eq. (10) simplifies to

$$\langle\chi_k(t)|\chi_j(t)\rangle = \langle\chi_0|e^{-it[f_k-f_j]H^{\rm e}}|\chi_0\rangle, \tag{12}$$

which is the autocorrelation function in the environment of χ_0 under a k and j-dependent rescaled time evolution.

Experimental realizations with trapped atoms

Experimental setups which allow to realize such a scheme have been proposed in [10,11], based on a single cold ion in a trapping potential involving two different electronic states $|1\rangle$ and $|2\rangle$. The electronic states play the role of the "central system," while—in our terminology—the center-of-mass motion of the ion should be identified with the "environmental" degrees of freedom. Here, the dynamics of interest is the motion of the ion in the trap (eventually, one may wish to find out whether it corresponds to classically chaotic or integrable motion). The proposal is based on an initial state involving a coherent superposition of both internal states,

$$\Psi(0) = 2^{-1/2}(|1\rangle + |2\rangle) \otimes |\chi_0\rangle. \tag{13}$$

Here, $|\chi_0\rangle$ being the initial motional state of the ion, for instance a coherent state. The ion evolves in the trap potential for some time t under the influence of an internal state-dependent potential, as explained previously. Physically, this is achieved with the help of a constant or pulsed off-resonant laser field (ac Stark effect). After some time t, the coherence $\varrho_{12}^{\rm c}(t)$ may be measured using Ramsey techniques [14]. A

recent experiment [15] with laser cooled Cs atoms exposed to the gravitational field and pulses of a standing wave of off-resonant light is close to a realization of such concepts. The authors use two hyperfine levels as internal "central" system, and propose to measure fidelity decay in a chaotic system.

Finally, let us mention an experiment where a quantity closely related to an echo fidelity is measured through the loss of coherence in a "central system." In [16] the authors investigate ultra cold ^{85}Rb atoms in an optical dipole trap. Using our terminology, the "central system" consists of internal electronic levels, while the center-of-mass motion of the atoms plays the role of the "environment." Starting with the initial state of Eq. (13), and applying an additional π-pulse right in the middle of the time evolution, one obtains for the coherences in the "central system:"

$$\varrho^{\mathrm{c}}_{12}(t) = \langle \chi_0 | U_2^\dagger U_1^\dagger U_2 U_1 | \chi_0 \rangle \varrho^{\mathrm{c}}_{12}(0). \tag{14}$$

Here, the echo operator is replaced by a product of four evolution operators over half the time interval, $t/2$, while the phases originating from the evolution of the central system have canceled. This particular variant of the echo operator has the advantage that the echo-signal is insensitive to the dephasing of different motional eigenstates of the atoms. In the experiment, it allows to observe an echo, even though about 10^6 states are thermally populated. Ultimately, the decay of the response is related to the detuning of the trap laser with respect to the different hyperfine states of the atoms.

III. AMPLITUDE COUPLING BETWEEN HARMONIC OSCILLATORS

In this section we consider a particular dissipative system, namely the famous quantum optical damped harmonic oscillator. Both, central system and environment consist of harmonic oscillators; the coupling is bilinear in annihilation and creation operators:

$$\begin{aligned} H &= H^{\mathrm{c}} + H^{\mathrm{e}} + H^{\mathrm{int}} \\ &= \hbar\Omega a^\dagger a + \sum_\lambda \hbar\omega_\lambda b_\lambda^\dagger b_\lambda + \sum_\lambda \hbar g_\lambda (a b_\lambda^\dagger + a^\dagger b_\lambda). \end{aligned} \tag{15}$$

Remarkably, despite so-called "amplitude coupling," this model allows for the correspondence between fidelity decay and decoherence. Moreover, the beautiful Paris decoherence experiment of a microwave field in a superconducting cavity [12] is adequately described by the Hamiltonian (15). In the light of the results to be shown, this decoherence experiment (for the central oscillator) may now also be interpreted as a "fidelity decay" experiment for the environment. A detailed theoretical description of the experiment may be found in [17].

As in the case of energy conserving coupling considered previously, we have to identify product state solutions of the dynamics. For Hamiltonian (15), they are given by products of coherent states. It is easy to see that with $|z\rangle = \exp\{-\frac{1}{2}|z|^2 + z a^\dagger\}|0\rangle$ for the central system and similarly defined coherent states $|\beta_\lambda\rangle$ for the oscillators of the environment, the product state

$$\begin{aligned} |\Psi(t)\rangle &= |z(t)\rangle \otimes |\beta_1(t)\rangle \otimes |\beta_2(t)\rangle \otimes \cdots \otimes |\beta_\lambda(t)\rangle \otimes \cdots \\ &\equiv |z(t)\rangle \otimes |B(t)\rangle \end{aligned} \tag{16}$$

is a solution of the Schrödinger equation. This holds true provided the coherent state labels follow the classical equations of motion:

$$i\partial_t z(t) = \Omega z(t) + \sum_\lambda g_\lambda b_\lambda(t), \tag{17}$$

$$i\partial_t \beta_\lambda(t) = \omega_\lambda \beta_\lambda(t) + g_\lambda z(t).$$

Assume, for simplicity (and also in very good agreement with experiment), a zero temperature environment such that all $\beta_\lambda(0)=0$. Formal integration leads to $\beta_\lambda(t) = -ig_\lambda \int_0^t ds e^{-i\omega_\lambda(t-s)} z(s)$. For the central system we find the effective equation

$$\dot{z}(t) + i\Omega z(t) + \int_0^t ds \alpha(t-s) z(s) = 0 \tag{18}$$

with the zero temperature bath correlation function $\alpha(t-s) = \Sigma_\lambda |g_\lambda|^2 e^{-i\omega_\lambda(t-s)}$. The actual experiment is well described by the Markov approximation which amounts to the replacement $\alpha(t-s) = \gamma\delta(t-s)$. Then $z(t) = \exp\{-i\Omega t - (\gamma/2)t\}$ displays the expected damped harmonic motion of the central oscillator. For the following argument, however, no such approximation is necessary.

We choose to investigate the fate of an initial macroscopic quantum superposition (Schrödinger cat) state of the central oscillator coupled to the environmental vacuum,

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}[|z_1(0)\rangle + |z_2(0)\rangle] \otimes |0\rangle. \tag{19}$$

Here, for simplicity, we assume $|z_1(0) - z_2(0)| \gg 1$ such that $\langle z_1(0) | z_2(0)\rangle \approx 0$ which simplifies the normalization in Eq. (19). Linearity demands that the total state evolves into the entangled state

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}|z_1(t)\rangle \otimes |B_1(t)\rangle + \frac{1}{\sqrt{2}}|z_2(t)\rangle \otimes |B_2(t)\rangle, \tag{20}$$

where we denote with $|B_i\rangle = |\beta_1^{(i)}\rangle \otimes |\beta_2^{(i)}\rangle \otimes \cdots \otimes |\beta_\lambda^{(i)}\rangle \otimes \cdots$ the environmental state corresponding to the initial state $|z_i(0)\rangle \otimes |0\rangle$. The coherent state labels in Eq. (20) evolve according to the classical equations (17) with initial conditions $\{z_1(0), \beta_\lambda^{(1)} = 0\}$ and $\{z_2(0), \beta_\lambda^{(2)} = 0\}$ respectively. The reduced density operator of the central system $\varrho^{\mathrm{c}} = \mathrm{Tr}_{\mathrm{e}} |\Psi\rangle\langle\Psi|$ is easily determined from the total state (20) and we find

$$\begin{aligned} \varrho^{\mathrm{c}}(t) = &\frac{1}{2}|z_1(t)\rangle\langle z_1(t)| + \frac{1}{2}|z_2(t)\rangle\langle z_2(t)| \\ &+ \frac{1}{2}\langle B_2(t)|B_1(t)\rangle |z_1(t)\rangle\langle z_2(t)| \\ &+ \frac{1}{2}\langle B_1(t)|B_2(t)\rangle |z_2(t)\rangle\langle z_1(t)|. \end{aligned} \tag{21}$$

Clearly, the time dependence of the coherence between the

superposed states is determined by the fidelity $\langle B_2(t)|B_1(t)\rangle$ of the corresponding environmental states.

As in the case of the energy-conserving coupling, the decaying fidelity may be interpreted as an echo fidelity. To see that, first notice that apart from an irrelevant phase with $\dot{\phi}_j(t)=-\frac{1}{2}\Sigma_\lambda g_\lambda[z_j(t)b_\lambda^{(j)}(t)+\text{c.c.}]$, the environmental states $|B_j(t)\rangle$ satisfy Schrödinger's equation with time dependent environment Hamiltonian

$$H_j^{\mathrm{e}}=\sum_\lambda \hbar\omega_\lambda b_\lambda^\dagger b_\lambda+\sum_\lambda \hbar g_\lambda(z_j(t)b_\lambda^\dagger+z_j^*(t)b_\lambda), \tag{22}$$

describing harmonic oscillations "driven" by the amplitude $z_j(t)$ of the damped central oscillator as determined from Eq. (18). Its initial value $z_j(0)$ arises from the initial state of the central system (19). Different initial coherent states $|z_j(0)\rangle$ lead to different Hamiltonians H_j^{e} in (22) and thus give different time evolutions of the environmental states. Similar to Eqs. (9)–(11) we may write

$$\begin{aligned}\varrho_{12}^{\mathrm{c}}(t)&=\langle B_2(t)|B_1(t)\rangle\varrho_{12}^{\mathrm{c}}(0)\\&=e^{-\mathrm{i}(\phi_1(t)-\phi_2(t))}\langle 0|\tilde{U}_0^\dagger(t)\tilde{U}(t)|0\rangle\varrho_{12}^{\mathrm{c}}(0),\end{aligned} \tag{23}$$

with the propagators arising from the Hamiltonians

$$\tilde{H}_0=H_1^{\mathrm{e}},$$

$$\tilde{H}=\tilde{H}_0+\{[z_2(t)-z_1(t)]\sum_\lambda \hbar g_\lambda b_\lambda^\dagger+\text{H.c.}\}. \tag{24}$$

The distance $|z_1-z_2|$ between the superposed coherent states determines the strength of the perturbation of the echo Hamiltonian (24). Thus, fidelity decay (and decoherence) become more rapid, as this distance increases. Assuming Markovian behavior and $\gamma t\ll 1$, our result reduces to the famous relation $|\varrho_{12}^{\mathrm{c}}(t)|^2=e^{-\gamma t|z_1(0)-z_2(0)|^2}|\varrho_{12}^{\mathrm{c}}(0)|^2$ [6,7,18].

IV. SITUATIONS WHERE PRODUCT STATE SOLUTIONS ARE ONLY APPROXIMATE

As the previous examples have shown, the relation between decoherence in the central system and fidelity decay in the environment works nicely, whenever it is possible to find product-state solutions of the coupled dynamics. In general, this will not be possible. Approximate product state solutions lead to approximate pure state solutions of the reduced dynamics and therefore to the concept of "robust" or "pointer" states [6,7,18]. Thus, if pointer states may be identified, the decoherence-fidelity relation will be satisfied in an approximate sense. A detailed discussion is beyond the scope of the current paper. The following short-time analysis of decoherence, however, allows for the desired relation in a very common situation.

Short time approach to decoherence

As the "distance" between superposed quantum states grows, decoherence may become very rapid. This observation is the starting point of a general short-time approach to decoherence recently developed [19–21]. Let us briefly sketch the main ideas.

Consider a central system with Hamiltonian H^{c} coupled to the environment (Hamiltonian H^{e}) through an interaction of the form

$$H^{\mathrm{int}}=S\otimes V, \tag{25}$$

where $S(V)$ is some operator in the Hilbert space of the central system (of the environment). Typically, the environmental part consists of contributions of many independent degrees of freedom, $V=\Sigma_\lambda V_\lambda$, but this is not of importance here. Decoherence in the central system will be most effective for initial states with largely different expectation values of S. In the famous quantum Brownian motion case [6,7,22], for instance, we have $H^{\mathrm{c}}=p^2/2m+V^{\mathrm{c}}(q)$, and $S=q$, the position operator.

In analogy to the oscillator case we assume an initial state of the form

$$|\Psi(0)\rangle=\frac{1}{\sqrt{2}}(|s\rangle+|s'\rangle)\otimes|B(0)\rangle, \tag{26}$$

where $|s\rangle$, $|s'\rangle$ and s, s' are eigenstates and corresponding eigenvalues of the operator S. In [19–21] it is argued that if $|s-s'|$ is large enough, decoherence may be so rapid as to outrun any dynamics induced by the Hamiltonian H^{c} of the central system. Thus, for these short times, H^{c} may be dropped entirely and the total Hamiltonian reads

$$H\approx S\otimes V+H^{\mathrm{e}}. \tag{27}$$

In this short-time approximation, eigenstates of S are conserved. Thus, we are essentially in the "dephasing" situation, discussed in Sec. II. Again, one finds product state solutions of the Schrödinger equation, here of the form

$$|\Psi(t)\rangle=|s\rangle\otimes|B_s(t)\rangle. \tag{28}$$

The environmental evolution is generated by the Hamiltonian

$$H_s^{\mathrm{e}}=H^{\mathrm{e}}+sV. \tag{29}$$

For the environmental dynamics, the eigenvalue s plays the role of a coupling strength to the "potential" V. Thus, for times shorter than any time scale induced by the central Hamiltonian H^{c}, the solution of the Schrödinger equation for the initial state (26) will be

$$|\Psi(t)\rangle=\frac{1}{\sqrt{2}}|s\rangle\otimes|B_s(t)\rangle+\frac{1}{\sqrt{2}}|s'\rangle\otimes|B_{s'}(t)\rangle. \tag{30}$$

The reduced density operator of the central system follows similarly to expression (21) and we conclude that the coherence between the states $|s\rangle$ and $|s'\rangle$ is given by the overlap

$$\begin{aligned}\varrho_{ss'}^{\mathrm{c}}(t)&=\langle s|\varrho^{\mathrm{c}}(t)|s'\rangle=\langle B_{s'}(t)|B_s(t)\rangle\varrho_{ss'}^{\mathrm{c}}(0)\\&=\langle B(0)|\tilde{U}_0^\dagger(t)\tilde{U}(t)|B(0)\varrho_{ss'}^{\mathrm{c}}(0).\end{aligned} \tag{31}$$

Here, the propagators correspond to the environment Hamiltonians

$$\tilde{H}_0 = H^e + s'V \text{ and } \tilde{H} = \tilde{H}_0 + (s - s')V. \quad (32)$$

Decoherence in ϱ^c may thus be interpreted as an echo fidelity with a perturbation proportional to the difference of the eigenvalues $s-s'$ of the initially superposed states. We recall and stress that by self-consistency, this simple short-time result is valid only as long as it predicts decoherence (fidelity decay) times that are short compared to "system" time scales induced by the central Hamiltonian.

V. CONCLUSIONS AND OUTLOOK

We have analyzed the connection between decoherence of a central system and fidelity decay in the environment for a variety of situations. This connection can be established easily if the energy of the central system is conserved (i.e., dephasing) as also discussed in [10,11]. Here, we have extended these ideas to more general situations. Interestingly, we have been able to show that even in the case of dissipation (amplitude coupling) a similar relation holds. Moreover, short time decoherence can be interpreted along these lines. Generally speaking, for our argument to be valid it is crucial that the Hamiltonian of the composite system allows for (approximate) product state solutions as time evolves. Then, the superposition of two such solutions allows to interpret the decoherence manifest in the off-diagonal matrix element of the reduced density matrix of the central system as a fidelity decay in the environment and vice versa. It is remarkable that properties of unitary time evolution in the environment and the non-unitary evolution of coherences become related.

Experiments based on these ideas can give important information about the stability of the unperturbed (environmental) Hamiltonian—a fact which might also be relevant for quantum information processing.

The connection between decoherence and fidelity decay can always be established whenever pointer states of the central system can be found. Then the factorization, essential to our argument, is valid for fairly long times. Our results highlight a beautiful complementarity: decoherence between pointer states may be interpreted as an act of measurement by the environment on the central system. The "collapsed" state of the central system may then be inferred from the environment. When measuring fidelity decay via decoherence, information about the environmental dynamics is extracted via observations on the central system. Implications of this connection will have to be studied in future work.

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