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LETTER TO THE EDITOR

Ruelle resonances in quantum many-body dynamics

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Abstract

We define a quantum Perron–Frobenius master operator over a suitable normed space of translationally invariant states adjoint to the quasi-local C^* algebra of quantum lattice gasses (e.g. spin chains), whose spectrum determines the exponents of decay of time correlation functions. The theoretical ideas are applied to a generic example of kicked Ising spin $\frac{1}{2}$ chains. We show that the 'chaotic eigenmodes' corresponding to leading eigenvalue resonances have fractal structure in the basis of local operators.

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It is well known that ergodic properties of classical dynamical systems, e.g. the rate of relaxation towards equilibrium, are connected to the spectral properties of Perron–Frobenius transfer operators (PFTO) on *appropriate* function spaces of phase space distributions [1, 2]. From a rigorous point of view, this connection has been established only for certain classes of 1D mappings and general Anosov systems [1]. However, there are many heuristic theoretical results and numerical experiments in the physics literature supporting the general applicability of the Ruelle resonance spectrum [3], more recently also in systems with mixed phase space [4]. We know that isolated bounded quantum systems can exhibit genuinely mixing behaviour (unique approach to equilibrium) only in the semiclassical or thermodynamic limit where the spectrum of the unitary (Schrödinger) propagator may become continuous. Recently, there were some attempts to define effective non-unitary quantum PFTO based on quantum–classical correspondence at small values of effective $\hbar$ [5, 6]. The spectrum of such quantum PFTO becomes the usual Ruelle spectrum of classical PFTO in the semiclassical limit $\hbar \to 0$, and can explain exponents of decay of quantum and classical correlation functions.

In this paper, we address the question whether one can define quantum PFTO in the thermodynamic limit of a many-body system without referring to any classical limit (which may not even exist, e.g. for Fermi lattices). We propose a definition of 'chaotic resonances', a non-commutative analogue of the Perron–Frobenius–Ruelle spectrum in infinite quantum lattice systems, providing a mechanism for the exponential decay of time correlations. The key idea in our proposal is a physical mechanism analogous to the phase space coarse graining

in classical and semiclassical approaches, namely we shall consider projections to finite dimensional spaces of local observables of finite but increasing order of locality. After a brief general discussion, we specialize on the properties of PFTO of a periodically kicked Ising spin $\frac{1}{2}$ chain. We show that the leading eigenvalues of PFTO inside the unit circle correspond to the exponential correlation decay, and that the structure of the corresponding eigenvectors is fractal in the basis of local density operators. A related spectral approach to quantum relaxation has been undertaken in [7].

We consider a quantum system living on a lattice[1] $\mathbb{Z}$ with each point $x \in \mathbb{Z}$ being associated with an N-level Hilbert space $\mathfrak{H}_x$. Let $\ell = \{\ell_1, \ell_1 + 1, \ldots, \ell_2\}$, $\ell_1 \leqslant \ell_2$, be a *sublattice* of $|\ell| := \ell_2 - \ell_1 + 1$ points, to which one assigns a *local* C^* algebra $\mathfrak{A}_\ell$ of bounded operators over $N^{|\ell|}$-dimensional space $\bigotimes_{x\in\ell} \mathfrak{H}_x$. Using the limit *with inclusion* $\ell \to \mathbb{Z}$ and *closure* in the operator norm $\|A\|$ one defines a standard *quasi-local* C^* operator algebra $\mathfrak{A}$ [8]. Let $g_j(x) = \mathbb{1}_{\{-\infty,\ldots,x-1\}} \otimes g_j \otimes \mathbb{1}_{\{x+1,\ldots,\infty\}}$, $j = 0, \ldots, N^2 - 1$ be the generators of $\mathfrak{A}_{\{x\}} \equiv GL(N)$, such that $g_0 = \mathbb{1}$, $\operatorname{Tr} g_j^\dagger g_k = N\delta_{jk}$, and $\mathcal{S}_m$ be the set of multi-indices $\boldsymbol{s} = (s_1, s_2, \ldots, s_m)$ with $s_1, s_m \in \{1, \ldots, N^2 - 1\}$ and $s_j \in \{0, \ldots, N^2 - 1\}$ for $1 < j < m$, and $\mathcal{S} = \bigcup_{m=1}^{\infty} \mathcal{S}_m$. The set of unimodal operators

$$G_{\boldsymbol{s}}(x) = g_{s_1}(x) g_{s_2}(x+1) \cdots g_{s_{l(\boldsymbol{s})}}(x + l(\boldsymbol{s}) - 1) \tag{1}$$

$\|G_{\boldsymbol{s}}(x)\| = 1$, for all $x \in \mathbb{Z}$ and $\boldsymbol{s} \in \mathcal{S}$, where $l(\boldsymbol{s}) = m$ if $\boldsymbol{s} \in \mathcal{S}_m$, together with an identity operator $\mathbb{1}$, form a *complete* basis of $\mathfrak{A}$, namely any $A \in \mathfrak{A}$ can be uniquely written as $A = a_0\mathbb{1} + \sum_{x\in\mathbb{Z}} \sum_{\boldsymbol{s}\in\mathcal{S}} a_{\boldsymbol{s}}(x) G_{\boldsymbol{s}}(x)$ for some $a_0, a_{\boldsymbol{s}}(x) \in \mathbb{C}$.

We introduce two essential automorphisms of the operator algebra $\mathfrak{A}$: *the shift automorphism* $\mathbf{S}$ simply translates the lattice by one site, namely $\mathbf{S}G_{\boldsymbol{s}}(x) := G_{\boldsymbol{s}}(x+1)$. *The time automorphism* $\mathbf{T}$ performs a finite-time ('one step') evolution generated by some Hamiltonian H. If H is autonomous then $\mathbf{T} = \mathrm{e}^{\mathrm{i}\tau \mathrm{ad}H}$ where $(\mathrm{ad}H)A := [H, A] = HA - AH$, or more generally, if H is periodically time dependent $H(t+\tau) = H(t)$ generating the unitary Floquet map U then $\mathbf{T}A = U^*AU$. Generally, existence of time automorphism is a separate and difficult problem (see [8]); here we will later apply our thinking to a model where the existence of $\mathbf{T}$ is obvious. We assume that our Hamiltonian is *translationally invariant* (TI) so the time and shift automorphisms commute, $\mathbf{TS} = \mathbf{ST}$.

One should imagine that $\mathbf{T}$ is just a quantum analogue of the Koopman operator which propagates observables, and is unitary in the operator topology, i.e. $\|\mathbf{T}A\| = \|A\|$ for any $A \in \mathfrak{A}$. In classical mechanics, PFTO is Koopman's adjoint propagating the phase space distributions, i.e. continuous linear functionals on the space of observables. With this analogy in mind, we consider time propagation in the space $\mathfrak{A}^*$ of continuous (i.e. bounded) linear functionals over $\mathfrak{A}$, including the convex subset of quantum states. Any element $\omega \in \mathfrak{A}^*$ is completely specified by its $\mathbb{C}$-values on the basis, $\omega_0 = \omega(1)$, $\omega_{\boldsymbol{s}}(x) = \omega(G_{\boldsymbol{s}}(x))$. It can be written in terms of a sequence of 'density operators' $\Omega_\ell(\omega) = \omega_0\mathbb{1} + \sum_{x,\boldsymbol{s}}^{\{x,x+l(\boldsymbol{s})-1\}\subset\ell} \omega_{\boldsymbol{s}}(x) G_{\boldsymbol{s}}(x)$ as $\omega(A) = \lim_{\ell\to\mathbb{Z}} \langle \Omega_\ell(\omega) A\rangle = \omega_0 a_0 + \sum_{\boldsymbol{s},x} \omega_{\boldsymbol{s}} a_{\boldsymbol{s}}$, where $\langle B\rangle := \lim_{\ell\to\mathbb{Z}} N^{-|\ell|} \operatorname{Tr}_\ell B$ is a *tracial state* (infinite temperature Gibbs state). This is due to orthonormality of the basis, $\langle G^*_{\boldsymbol{s}'}(x') G_{\boldsymbol{s}}(x)\rangle = \delta_{\boldsymbol{s},\boldsymbol{s}'}\delta_{x,x'}$, $\langle \mathbb{1} G_{\boldsymbol{s}}(x)\rangle = 0$. However, a formal inclusion limit $\Omega(\omega) = \lim_{\ell\to\mathbb{Z}} \Omega_\ell(\omega)$ will generally not converge inside quasi-local algebra $\mathfrak{A}$. As for general properties of coefficients of arbitrary $A \in \mathfrak{A}$, $\omega \in \mathfrak{A}^*$ we note that $a_{\boldsymbol{s}}(x)$ must be bounded in l_2 norm, since $\|A\|_2^2 := |a_0|^2 + \sum_{x,\boldsymbol{s}} |a_{\boldsymbol{s}}(x)|^2 < \|A\|^2 < \infty$, while $\omega_{\boldsymbol{s}}(x)$ must only be bounded in l_∞ norm, since $\infty > \|\omega\| := \sup_{A\in\mathfrak{A}} |\omega(A)|/\|A\| \geqslant \|\omega\|_\infty := \sup_{x,\boldsymbol{s}}\{|\omega_0|, |\omega_{\boldsymbol{s}}(x)|\}$.

[1] Here we consider one-dimensional lattices though we believe our discussion should be easily generalized to higher dimensions.

In the following, we will limit our discussion on the subspace $\mathfrak{T} \subset \mathfrak{A}^*$ of TI states perpendicular to the trivial equilibrium state, $\mathfrak{T} = \{\omega \in \mathfrak{A}^*; \omega \circ \mathbf{S} = \omega, \omega_0 = 0\}$. From the translational invariance, it follows that coefficients should be independent of x, $\omega_s(x) \equiv \omega_s$, so the density operator is of the form $\Omega(\omega) = \sum_s \omega_s Z_s$ in terms of the operator basis $Z_s := \sum_x G_s(x)$. The space $\mathfrak{T}$ is obviously invariant under any TI time automorphism $\mathbf{T}$, i.e. if $\omega \in \mathfrak{T}$ then $\omega \circ \mathbf{T} \in \mathfrak{T}$. It is suitable to define a natural metric on the space $\mathfrak{T}$ of TI states, namely for $\omega, \theta \in \mathfrak{T}$,

$$(\omega|\theta) = \lim_{\ell \to \mathbb{Z}} \frac{1}{|\ell|} \langle \Omega_\ell^*(\omega) \Omega_\ell(\theta) \rangle = \sum_{s \in \mathcal{S}} \omega_s^* \theta_s. \tag{2}$$

We note that the Hilbert space of TI operators $\Omega(\omega)$ with $\|\omega\|_2^2 := (\omega|\omega) < \infty$ is a *dynamical Lie algebra* of spatially extended *pseudo-local* observables studied in [10].

Now we define a quantum PFTO as the *adjoint* time automorphism $\hat{\mathbf{T}}$ on $\mathfrak{T}$,

$$(\hat{\mathbf{T}}\omega)(A) = \omega(\mathbf{T}A). \tag{3}$$

However, the master operator $\hat{\mathbf{T}}$ is strictly unitary, $\|\hat{\mathbf{T}}\omega\| = \sup_A |\omega(\mathbf{T}A)|/\|A\| = \|\omega\|$, if we insist on continuity of ω with respect to test operators A from the entire quasi-local algebra $\mathfrak{A}$. Instead, one should consider a different (smaller) space of test operators A so that $\hat{\mathbf{T}}$ has a possible point spectrum inside the unit circle. Rigorous analysis of the required operator space leads to difficult mathematical problems, so we suggest here a different, practical solution. Namely, for an explicit construction of PFTO we propose a kind of Ulam finite-rank approximation. Let $\hat{\mathbf{P}}_r$ denote a complete sequence of finite-rank orthogonal projections from $\mathfrak{T}$ onto its finite-dimensional subspaces $\mathfrak{T}_r = \{\omega \in \mathfrak{T}; \omega(G_s(x)) = 0 \text{ if } l(s) > r\}$, $\dim \mathfrak{T}_r = (N^2 - 1)N^{2r-2}$. We define a sequence of operators $\hat{\mathbf{T}}_r = \hat{\mathbf{P}}_r \hat{\mathbf{T}} \hat{\mathbf{P}}_r$ which may be represented with finite-dimensional matrices $T^{(r)}_{s,s'}$ as $(\hat{\mathbf{T}}_r\omega)_s = \sum_{s'}^{l(s)\leqslant r} T^{(r)}_{s,s'}\omega_{s'}$. Although $\lim_{r\to\infty} \hat{\mathbf{T}}_r$ cannot converge in master operator topology (but may converge in ω-topology), we conjecture that there may be a part of the eigenvalues and eigenvectors of $\hat{\mathbf{T}}_r$ which converges with growing r to, what we call, the quantum Ruelle resonance spectrum. We note that the space $\mathfrak{T}_r$ may be considered as an analogy to the space of coarse grained phase-space densities with resolution $1/r$.

One of the most straightforward applications of the Ruelle resonance spectrum is the decay of time correlations. Let $\{\mathrm{e}^{-q_n}\}$ be a point spectrum of $\hat{\mathbf{T}}$, non-degenerate by assumption, and $\{\theta_n^{\mathrm{R}}\}, \{\theta_n^{\mathrm{L}}\}$ the corresponding right and left eigenvectors, $\hat{\mathbf{T}}\theta_n^{\mathrm{R}} = \mathrm{e}^{-q_n}\theta_n^{\mathrm{R}}$, $\hat{\mathbf{T}}^\dagger\theta_n^{\mathrm{L}} = \mathrm{e}^{-q_n^*}\theta_n^{\mathrm{L}}$, defined by the limit $r \to \infty$ from the spectral representations of $\hat{\mathbf{T}}_r$. Then, for any pair of states $\eta, \omega \in \mathfrak{T}$, or extended TI observables $\Omega(\eta), \Omega(\omega)$, the time correlation function $(\eta|\omega(t)) = \lim_{\ell\to\mathbb{Z}}\langle \Omega_\ell(\eta) U^t \Omega_\ell(\omega) U^{-t}\rangle$ has the asymptotic behaviour (see, e.g., [2], p 90):

$$(\eta|\omega(t)) \to \sum_n w_n \,\mathrm{e}^{-q_n t} \qquad w_n = \frac{\left(\eta\middle|\theta_n^{\mathrm{R}}\right)\left(\theta_n^{\mathrm{L}}\middle|\omega\right)}{\left(\theta_n^{\mathrm{L}}\middle|\theta_n^{\mathrm{R}}\right)}. \tag{4}$$

It is interesting to point out that the denominator $\left(\theta_n^{\mathrm{L}}\middle|\theta_n^{\mathrm{R}}\right)$ may be *finite*, although we have $\left\|\theta_n^{\mathrm{L,R}}\right\|_2 = \infty$ for any eigenvalue away from the unit circle, $\mathrm{Re}\, q_n \neq 0$, since $\hat{\mathbf{T}}$ is unitary in the $\|.\|_2$ norm. If there is a gap and the spectrum $\{\mathrm{e}^{-q_n}\}$ lies strictly inside a unit circle, apart from the possible exception of a non-degenerate equilibrium state, then the quantum many-body system is (exponentially) mixing, whereas if there are non-trivial eigenvalues on the unit circle then the system is non-mixing. The latter happens in generic completely integrable many-body systems [9], where the infinite set of conservation laws Q_n [11] in our language corresponds to infinitely degenerate eigenvalue 1 of PFTO, $\hat{\mathbf{T}}Q_n = Q_n$.

Let us illustrate and validate our ideas by a specific example. We consider the kicked Ising (KI) spin $\frac{1}{2}$ chain, kicked periodically with a tilted magnetic field [12], having a

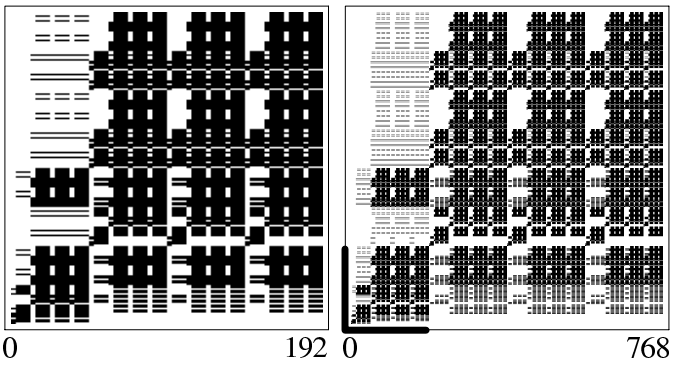

Figure 1. Non-vanishing matrix elements (black) of KI-PF matrix for $r = 4$ (left), 5 (right) in the code-ordered basis.

Floquet map $U = \mathrm{e}^{-\mathrm{i}H_1}\,\mathrm{e}^{-\mathrm{i}H_0}$ with $H_1 = J\sum_{x\in\mathbb{Z}}\sigma_1(x)\sigma_1(x+1)$, $H_0 = \sum_{x\in\mathbb{Z}} h_{\mathrm{x}}\sigma_1(x) + h_{\mathrm{z}}\sigma_3(x)$. Here, $N = 2$, and the generators of $GL(2)$ are the identity $g_0 = \mathbb{1}$ and Pauli matrices, $g_j = \sigma_j$, $j = 1, 2, 3$. It has been established that the KI model is non-trivially integrable for transverse field $h_{\mathrm{x}} = 0$ [9], while rich behaviour, ranging from non-ergodic to mixing dynamics, has been suggested in non-integrable cases of a tilted field [12]. The time evolutions in spaces $\mathfrak{A}$ and $\mathfrak{T}$ are given by

$$\mathbf{T} = \mathrm{e}^{\mathrm{i}\,\mathrm{ad}H_0}\,\mathrm{e}^{\mathrm{i}\,\mathrm{ad}H_1} \tag{5}$$

and (3). The KI model has a convenient feature, namely $\hat{\mathbf{T}}\mathfrak{T}_r \subset \mathfrak{T}_{r+1}$, so that $\hat{\mathbf{T}}_r$ can be computed *exactly*, since the adjoint propagators $\mathrm{e}^{z\,\mathrm{ad}H_j}$ can only increase the order r of locality by j, $j \in \{0, 1\}$. We have devised a *divide-and-conquer* algorithm (details will be published elsewhere) for performing *fast* computation of the transformation $\mathrm{e}^{\mathrm{i}\,\mathrm{ad}H_j}\sum_s^{l(s)\leqslant r} v_s Z_s = \sum_s^{l(s)\leqslant r+1} v'_s Z_s$, and hence the complete truncated matrix $T^{(r)}$, in $\mathcal{O}(r4^r)$ computer operations. Numerical computation of the full spectrum of $T^{(r)}$ is limited to the maximal order $r = 7$ by the diagonalization procedure, while the leading eigenvalue resonance can be determined up to $r = 12$ using an efficient power method. It is suitable to order the multi-indices $\boldsymbol{s} = \boldsymbol{s}(c)$ by increasing value of a unique code $c = \sum_{m=1}^{l(\boldsymbol{s})} s_m 4^{m-1}$. The allowed code c should not be divisible by 4 since $s_1 \neq 0$, and its length $l(\boldsymbol{s})$ is the position of its most significant non-zero digit. Therefore, the KI-PF matrix $T_{c'c} \equiv T^{(r)}_{\boldsymbol{s}(c')\boldsymbol{s}(c)}$ has a geometric block-band structure, namely $T_{c'c} = T_{cc'} = 0$ if $c' > 16c$. One can show that the matrix $T_{c'c}$ has a fractal structure, i.e. self-similarity upon scaling of row/column indices by factor 4, or increasing r by 1 (see figure 1). Note that the structure of the KI-PF matrix $T_{c'c}$ does not depend on the values of parameters $J, h_{\mathrm{x}}, h_{\mathrm{z}}$, except in the integrable planes $h_{\mathrm{x}} = 0$, and $h_{\mathrm{z}} = 0$, where additional matrix elements vanish.

KI-PFTO has a good parity, $\hat{\mathbf{R}}\hat{\mathbf{T}} = \hat{\mathbf{T}}\hat{\mathbf{R}}$, defined as $(\hat{\mathbf{R}}\omega)_{(s_1,s_2,\ldots,s_r)} = \omega_{(s_r,s_{r-1},\ldots,s_1)}$, so the spectrum of $\hat{\mathbf{T}}$ can be labelled as even/odd w.r.t. eigenvalue ± 1 of the parity operation $\hat{\mathbf{R}}$. The symmetry $\hat{\mathbf{R}}$ has been used to roughly halve the dimension of the matrices $T^{(r)}$. In figure 2 we show spectra of the truncated KI-PFTO $\hat{\mathbf{T}}_r$ for increasing order $r = 5, 6, 7$. We consider two cases of parameter values: (a) strongly non-integrable $J = 0.7, h_{\mathrm{x}} = 0.5, h_{\mathrm{z}} = 1.1$, and (b) integrable $J = 0.7, h_{\mathrm{x}} = 0.0, h_{\mathrm{z}} = 1.1$. In case (a) we find converged eigenvalues lying well inside the unit circle (see table 1 for the leading two resonances), while in case (b) we have a multiply degenerate eigenvalue 1 corresponding to local invariants of motion. We have systematically scanned a large portion of the parameter space $(J, h_{\mathrm{x}}, h_{\mathrm{z}})$ and found a rich variety of behaviour, quite importantly also *a finite proportion* of non-integrable cases where

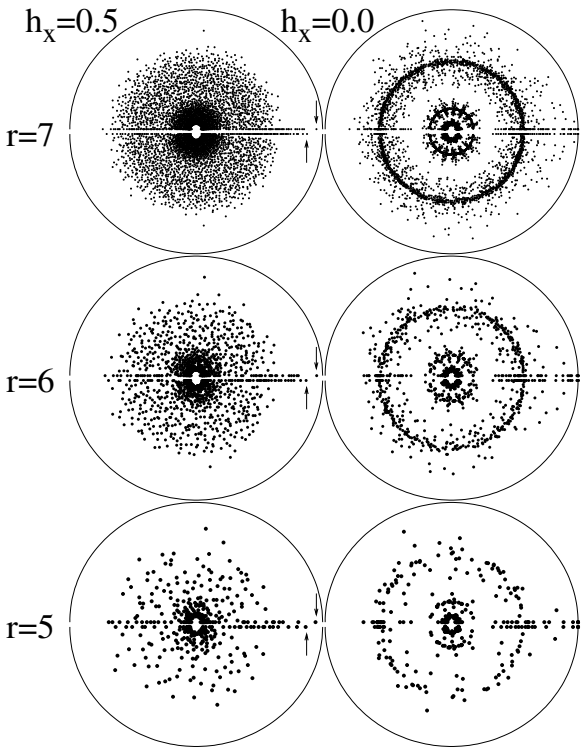

Figure 2. The spectra of truncated KI-PF operators $\mathbf{T}_r$, for $r = 5, 6, 7$ in strongly non-integrable (left) and integrable (right) cases (see text), lying inside complex unit circles (thin curves). The points in the upper/lower unit semi-discs correspond to positive/negative parity eigenvectors. Arrows point at the converged position of the leading eigenvalue e^{-q_1}.

Table 1. The convergence of two leading resonances together with their weights with respect to magnetization $M = \Omega(\mu)$.

n	r	q_n	$w_n(\mu)$
1	6	0.048 791	0.048 10
	9	0.047 741	0.046 76
	12	0.047 074	0.046 59
2	5	$0.2413 - 1.474\mathrm{i}$	$(5.25 + 10.31\mathrm{i}) \times 10^{-3}$
	6	$0.1903 - 1.491\mathrm{i}$	$(0.453 + 4.444\mathrm{i}) \times 10^{-3}$
	7	$0.1904 - 1.499\mathrm{i}$	$(0.294 + 4.679\mathrm{i}) \times 10^{-3}$

some eigenvalues e^{-q_n} converge onto the unit circle (most often $\to \pm 1$) thereby making the dynamics non-mixing or even non-ergodic (e.g. as in the 'intermediate case' reported in [12]). In the following, we concentrate on the ergodic and mixing case (a).

In figure 3 we compare the time autocorrelation of the magnetization $\mu_s = \delta_{s,(3)}$, $M = \Omega(\mu) = \sum_{x\in Z} \sigma_3(x) = Z_{(3)}$, computed from: (i) exact time evolution of finite lattices of length L with periodic boundary conditions (described in [12]), (ii) iteration of truncated KI-PF matrix $C_r(t) = \left(\mu \middle| \hat{\mathbf{T}}_r^t \mu\right)$ and (iii) asymptotics (4) based on the leading eigenvalue resonance. We conclude that convergence of KI-PF matrix results is indeed very fast, since the curves $C_r(r)$ for $r = 6, 7, 8, \ldots$, are practically indistinguishable. On the other hand, simulations on

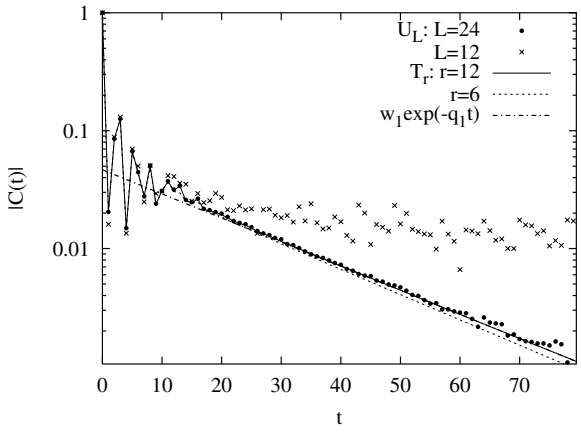

Figure 3. Correlation function $C(t) = (\mu|\mu(t))$, in the mixing case $h_x = 0.5$, computed from finite system dynamics for different sizes L (symbols), and from truncated adjoint propagators of infinite systems (curves) for different orders r.

finite lattices are much more time consuming, and convergence with L is much slower. We show that the exponent, and the pre-factor, of the asymptotic decay are indeed given by the leading eigenvalue q_1 of $\hat{\mathbf{T}}$ and the corresponding weight $w_1(\mu)$. This description of relaxation to equilibrium can be systematically improved by including other resonances—stable eigenvalues of $\hat{\mathbf{T}}_r$. The convergence of the eigenvalues and weights of the two leading resonances with order r is shown in table 1. For any 'localized' observable/state, such as M, the numerical weights w_n are significant only for a small part of the eigenvalues. These correspond in the limit $r \to \infty$ to the point spectrum of $\hat{\mathbf{T}}$. For the majority of eigenvalues which correspond to the essential spectrum and whose distribution typically looks stationary with r, the weights w_n quickly tend to zero. In other words, for true eigenvectors the denominator of w_n (see equation (4)) is finite, i.e. the sum $\left(\theta_n^{\mathrm{L}}|\theta_n^{\mathrm{R}}\right) = \sum_{m=1}^{r} u_m$, where $u_m := \sum_{s}^{l(s)=m} \left(\theta_n^{\mathrm{R}}\right)_s^* \left(\theta_n^{\mathrm{L}}\right)_s$, converges as $r \to \infty$ since u_m are fast decaying, so it can be normalized by $\sum_{m=1}^{\infty} u_m = 1$. On the other hand, for 'extended' eigenvectors corresponding to the essential spectrum coefficients u_m do not decay with m (or even increase) so $\sum_{m=1}^{\infty} u_m = \infty$. Convergence and fast decay of the series u_m for the leading eigenvalue resonance of case (a) is shown in the inset of figure 4. However, convergence of $\left(\theta_n^{\mathrm{L}}|\theta_n^{\mathrm{R}}\right)$ does not mean that the expansion coefficients of, say the right, eigenvector $v_c = \left(\theta_n^{\mathrm{R}}\right)_{s(c)}$ will decay as $c \to \infty$. In fact, we know that $\sum_c |v_c|^2 = \infty$ if $\mathrm{Re}\, q_n > 0$. Instead we find, for well-converged resonances, that the structure of the coefficients v_c is typically fractal: for sufficiently large c we have statistical self-similarity

$$\overline{|v_c|} = \alpha \overline{|v_{4c}|} \tag{6}$$

where $\bar{\cdot}$ means averaging over a narrow range of code, which is a consequence of the fractal matrix structure (figure 1). This is illustrated in figure 4 where we show v_c for the leading resonance $n = 1$ with the estimated scaling exponent $\alpha \approx 1.56 \pm 0.05$. The corresponding left eigenvector has statistically similar behaviour.

An appropriate quantum Perron–Frobenius evolution operator for infinite quantum lattice systems has been proposed whose lack of unitarity provides a mechanism of quantum dissipation and relaxation to equilibrium in isolated conservative systems. We have constructed a numerical example of kicked Ising chain, where we have shown how the leading eigenvalue of the transfer operator and the corresponding left and right eigenvector explain the asymptotic

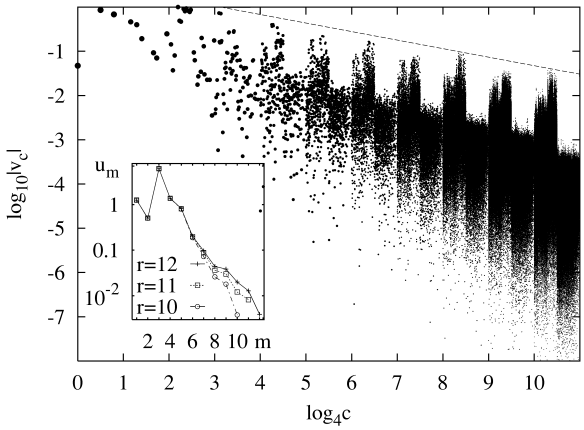

Figure 4. Fractal properties of expansion coefficients v_c of the right leading resonance $n = 1$ eigenvector computed for $r = 12$. The dashed line indicates the scaling $c^{-0.32}$. In the inset we demonstrate fast decay of u_m, for $m > 3$, at three different values of $r = 10, 11, 12$.

behaviour of time correlation function of arbitrary observables/states. We believe that these are generic results and that our method can also be applied in other, say time-independent, quantum lattices.

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