(QUANTUM) CHAOS AND HYDRODYNAMICS IN HOLOGRAPHIC AND WEAKLY COUPLED FIELD THEORIES

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Integrable and chaotic quantum dynamics: from holography to lattice

Bled

Based on 1710.00921, 1804.09182 and other work in progress with Sašo and Koenraad

CLASSICAL THERMALISATION

How does an isolated classical system thermalise?

Chaotic dynamics makes the system ergodic at long times

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Not every classical system thermalises:

- Integrable
- Integrable with non linear perturbations KAM

‘QUANTUM THERMALIZATION’

- Experiments on cold atomic gases have shown that closed quantum systems have a thermal spectrum

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related to the BH Information Paradox (BH radiation)

- What happens to the BH S-matrix if I throw some particles before the emission?
- Chaos in the BH S-matrix?

Polchinski '15

⇒ Chaos might play a role also in the quantum regime

How can we define (and measure) quantum chaos?

Larkin and Ovchinnikov

HOW DO WE MEASURE QUANTUM CHAOS?

$$C(t,\vec{x}) = -\langle [W(t,\vec{x}),V(0)][W(t,\vec{x}),V(0)]\rangle$$

Intrinsically non local: Out-of-time correlator (OTOC)

In the semiclassical limit: $W(t) = q(t) \quad V = p$

$$[W(t),V(0)] \longrightarrow i\hbar\{q(t),p(0)\} = i\hbar\frac{\delta q(t)}{\delta q(0)}$$

For systems whose classical limit is chaotic

$$\delta q(t) = \delta q(0)e^{\lambda_L t} \qquad C(t) = \hbar^2 e^{2\lambda_L t}$$

This correlation becomes $O(1)$ at $t_* = \frac{1}{\lambda_L}log(1/\hbar)$

Kitaev

$$C(t,\vec{x}) \propto e^{2\lambda_L(t-|\vec{x}|/v_B)}$$

Butterfly velocity

INFORMATION SCRAMBLING IN BH AND THERMALISATION OF A QUANTUM SYSTEM

- Information is not lost, it is scrambled! Not accessible to local observables.
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Unitarity is preserved

- How fast does a BH scramble (hide/delocalize) its information?

Puzzle:

- Systems with conserved charges on a long time scale are described by hydro, which is dissipative: where is unitarity?

- Unitarity provides at least some conserved charges: norm of operators (emergent hydro)

$$tr(O^\dagger(t)O(t))$$

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How does diffusive Hydro (dissipative) emerge from unitary evolution (reversible)?

Scrambling is the answer!

Khemani, Vishwanath and Huse '17

CHAOS AND DIFFUSION

There seem to be an interplay between scrambling and diffusion but

Quantum chaos → Early time dynamics

Diffusion/hydro → Late time dynamics

How can they be related?

- Hartnoll (2014) proposed that diffusion in strongly coupled systems with no quasiparticles is set by

$$D = \frac{\hbar v^2}{k_B T}$$

$$\tau = \frac{\hbar}{k_B T}$$

Planckian time

Sachdev, Zaanen

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Sachdev, Zaanen

Insights from holography:

- Blake $\tau \propto \lambda_L^{-1}$ and $v = v_B$

- Charge diffusion in locally charge neutral QFT

$$D_c = \mathcal{C}\frac{v_B^2}{\lambda_L}$$

Blake '16

- Energy diffusion $D_T = \frac{z}{2z-2}\frac{v_B^2}{\lambda_L}$

Blake, Davison and Sachdev '17

Why are they related?

Bound on chaos

Under mild assumptions it can be shown that

$$\lambda_L \leq \frac{2\pi k_{\mathrm{B}} T}{\hbar}$$

Maldacena, Shenker and Stanford '15

Is it possible to derive a lower/upper bound on diffusion?

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

Kovtun, Son and Starinets '05

$$D \geq v^2 \frac{\hbar}{k_B T}$$

Hartnoll '15

$$D \leq v_{LR}^2 \tau_{eq}$$

Hartman, Hartnoll and Mahajan '17

Emergent light cone + diffusive transport

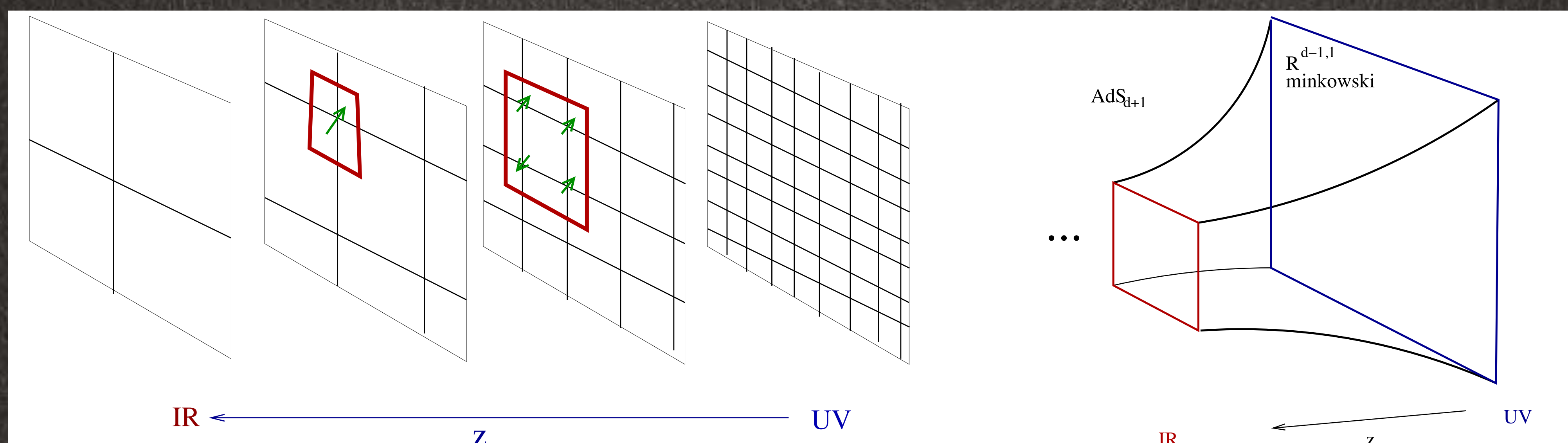
$$D \leq v_B^2 \tau_{eq}$$

Lucas '17

Many-body quantum chaos + diffusive transport

HOLOGRAPHIC CORRESPONDENCE

Classical gravity in asympt AdS spacetimes ⇄ Strongly coupled Large N Gauge theories



Maldacena, Witten, Gubser, Klebanov, Polyakov

$$Z_{CFT}[J] = e^{iS_{AdS}^{on-shell}}[\phi|\phi_{AdS}=J]$$

Finite temperature	⇄	BH temperature
Chemical potential		Boundary value of A_t
Global internal symmetries		Gauge symmetries
Linearised gravity dynamics		Energy dynamics

CHAOS IN HOLOGRAPHY

- The exponential growth of OTOC is encoded in the shockwave geometry

$$ds^2 = A(uv)dudv + V(uv)d\vec{x}_d^2 - A(uv)h(\vec{x})\delta(u)du^2$$

Shenker and Stanford '14
't Hooft, Sfetsos

- The EOM is

$$\left(\partial_i\partial_i - \frac{dV'}{A}\right)h(\mathbf{x}) \propto \frac{16\pi V(0)}{A(0)}Ee^{\frac{2\pi t_w}{\beta}}\delta(\mathbf{x})$$

But (from the dictionary): linearised gravity dynamics is <u>energy dynamics</u>

BLACK HOLES SCRAMBLING FROM HYDRODYNAMICS

Based on Grozdanov, Schalm and VS '17

- We construct a sound solution (spin 0 metric fluctuation)

⇩

It encodes all relevant shock dynamics

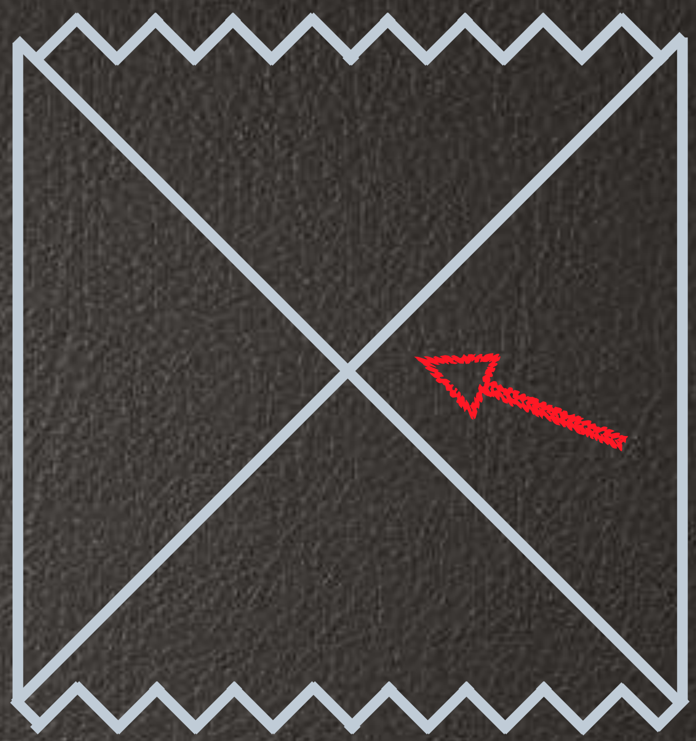
- This solution is encoded in the sound channel spectrum, analytically continued to $\mathrm{Im}\,\omega > 0 \quad k \in i\mathbb{R}$

- In holography it is possible to reconstruct the information about chaos (OTOC) by studying the E-E retarded Green's function

Grozdanov, Schalm and VS

We construct a sound solution

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + b(r)\left(dx^2 + dy^2 + dz^2\right)$$
$$- \left[f(r)H_1 dt^2 - 2H_2 dt dr + \frac{H_3 dr^2}{f(r)} + H_4 \left(dx^2 + dy^2\right) \right]$$



AdS_5 -Schwarzschild

With

$$H_1 = H_3 = \left(C_+ W_+(t,z,r) + C_- W_-(t,z,r)\right),$$
$$H_2 = \left(C_+ W_+(t,z,r) - C_- W_-(t,z,r)\right)$$

and

$$W_\pm(t,z,r) = e^{-i\omega\left[t \pm \int^r \frac{dr'}{f(r')}\right] + ikz} h_\pm(r),$$

Grozdanov, Schalm and VS

- EOMs imply a diffusion relation

$$G_{tr} = 0 \quad \Rightarrow \quad \omega_{\pm} \equiv \pm i\mathfrak{D}k^2 = \pm i\frac{1}{3\pi T}k^2 \,.$$

- Regularity at the horizon implies a constraint

$$\boxed{k^2 \equiv -m^2 = -6\pi^2 T^2}$$

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- This sound wave solution is a smeared shockwave:

$$\omega_{\pm} \equiv \mp i\lambda_L \,, \quad \lambda_L = 2\pi T \,,$$
$$v_B \equiv \left|\frac{\omega_{\pm}}{k}\right| = \sqrt{\lambda_L \mathfrak{D}} \,.$$

$$\delta(U) \to 1/U$$

$$U\partial_U \Delta(U) = -\Delta(U) \qquad U^2\partial_U^2 \Delta(U) = 2\Delta(U)$$

- It works at the linearised level

IS IT A SOUND MODE?

In holography sounds is related to the two lowest quasinormal frequencies

- We need to check if these modes are in the pole structure of

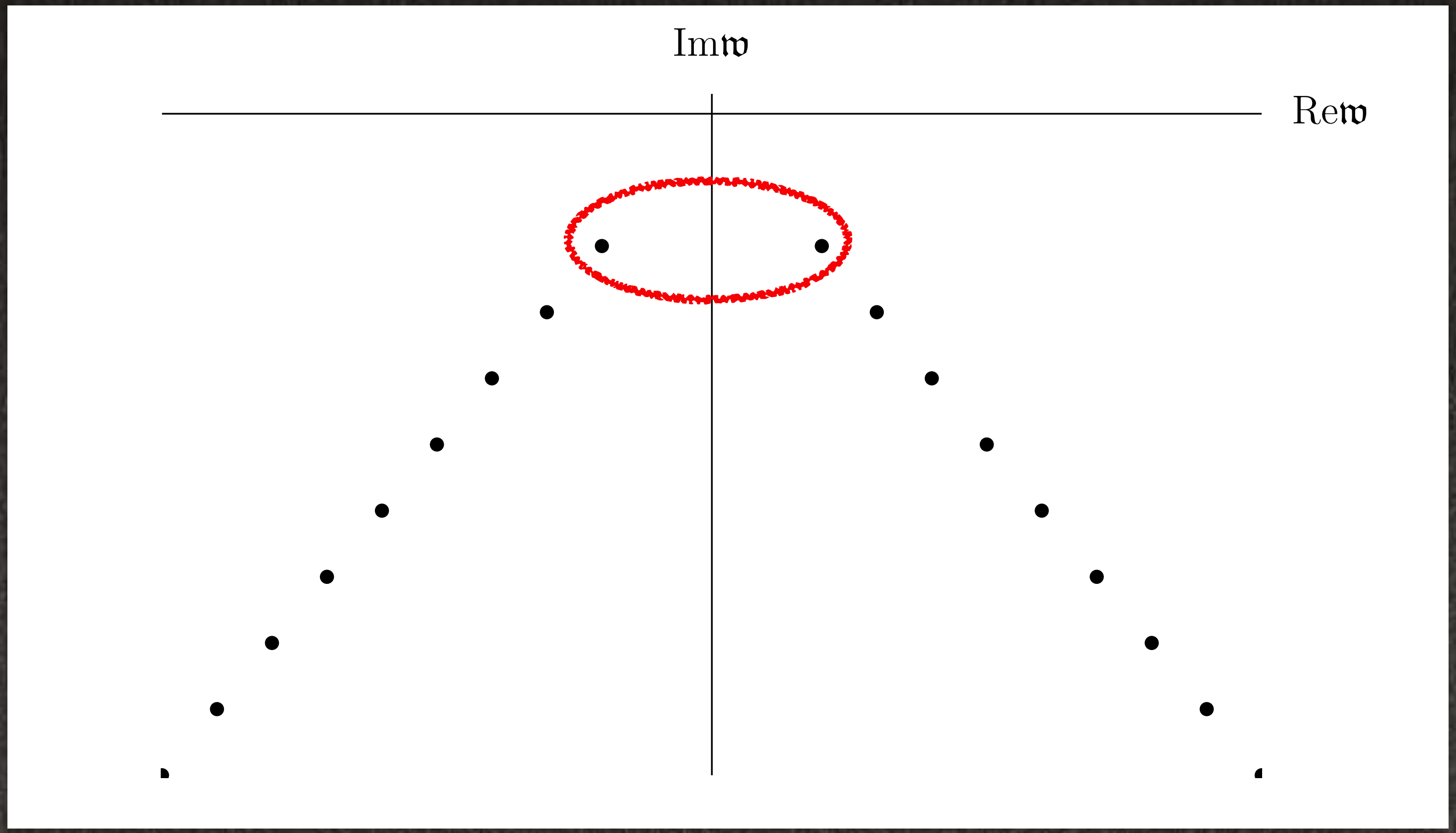
$$G^R_{T^{00}T^{00}}(\omega, k)$$

Numerically the solution is exact: the full series, not only the first terms!

Analytically possible in the hydro limit

$$\beta|\omega| \ll 1 \qquad \beta|k| \ll 1$$

$$\omega^*_{\pm}(k) \approx \pm v_s k - iDk^2 + \ldots$$



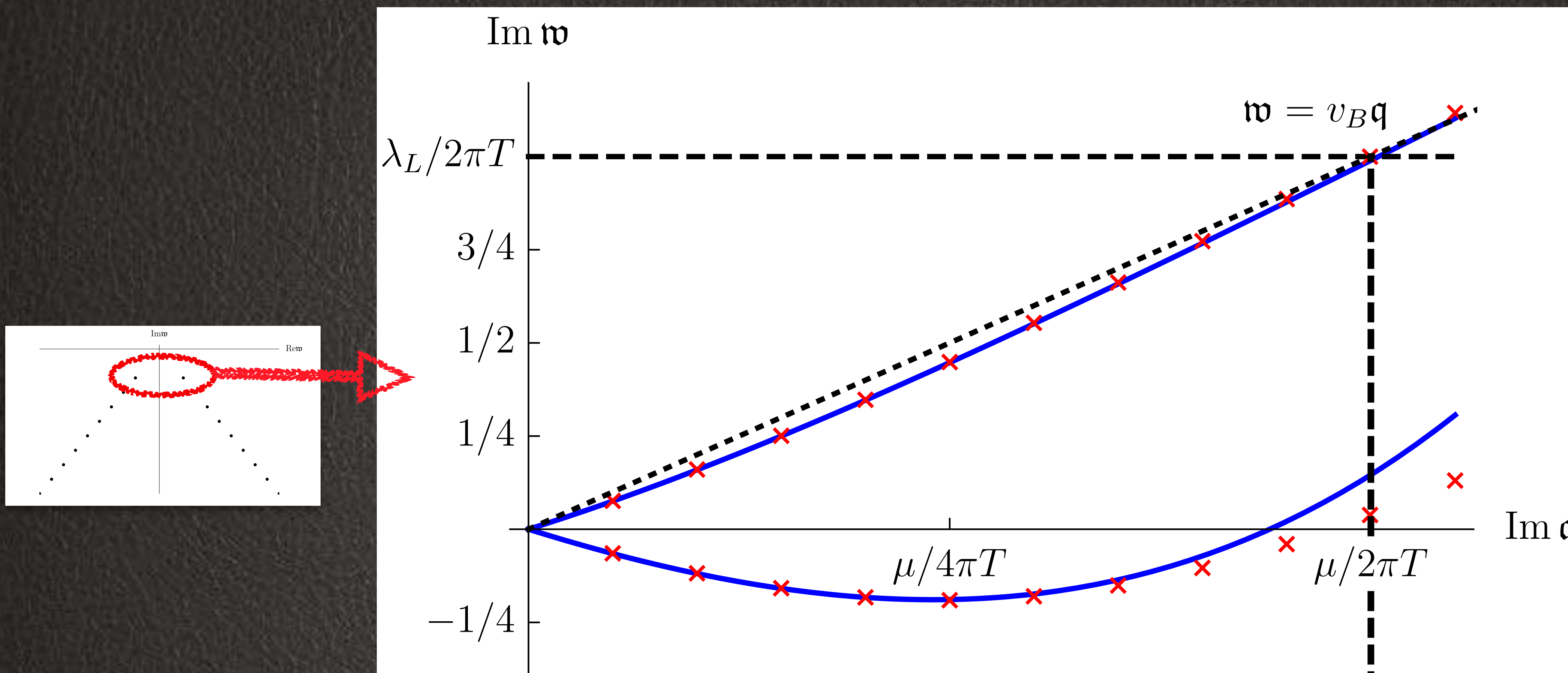
$$\omega^*_\pm(k) \approx \pm \sum_{n=0}^{\infty} \mathcal{V}_{2n+1} k^{2n+1} - i \sum_{n=0}^{\infty} \Gamma_{2n+2} k^{2n+2}$$

Real

- Sound modes (relaxation time)
- Penetration depth (relaxation length)
- Shockwave solution $\omega^*_\pm(k = i\mu) = i\lambda_L$

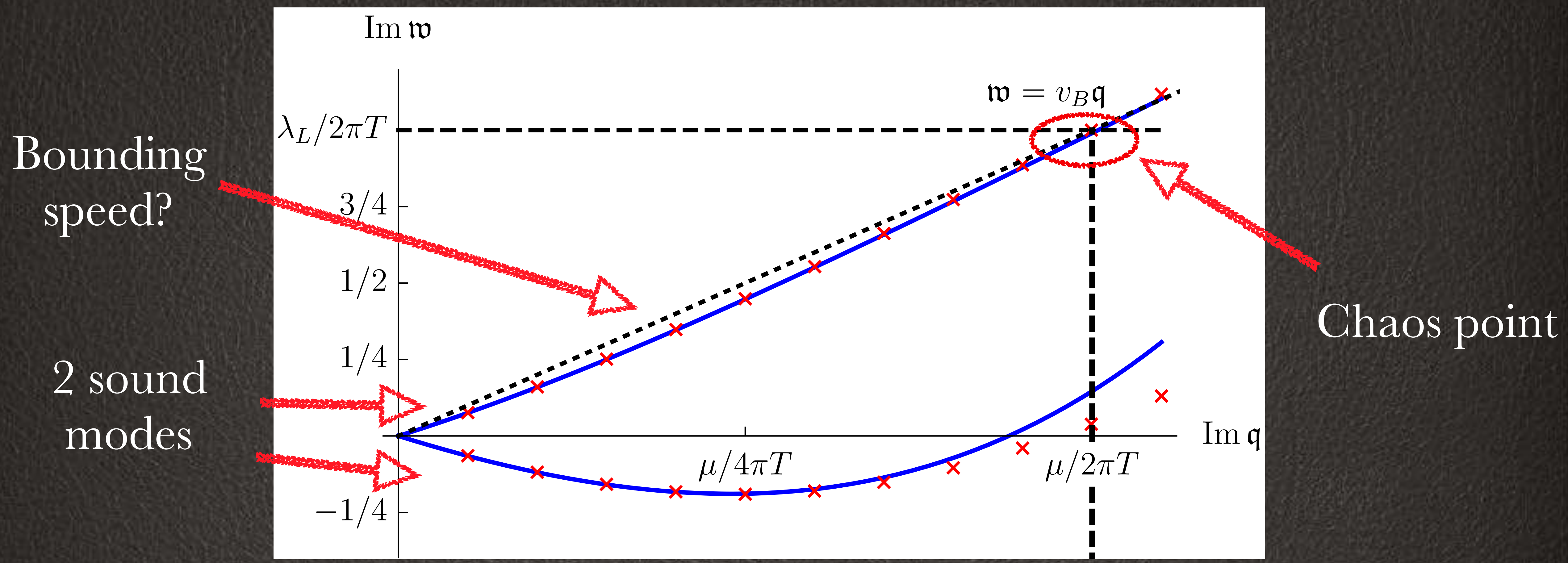
ω^*	k
Complex	Real
Real	Complex
Imaginary	Imaginary

QUANTUM CHAOS FROM HYDRO



- The information about chaos is recovered from the sound channel spectrum analytically continued to $\mathrm{Im}\,\omega$ and $\mathrm{Im}\,k$

QUANTUM CHAOS FROM HYDRO



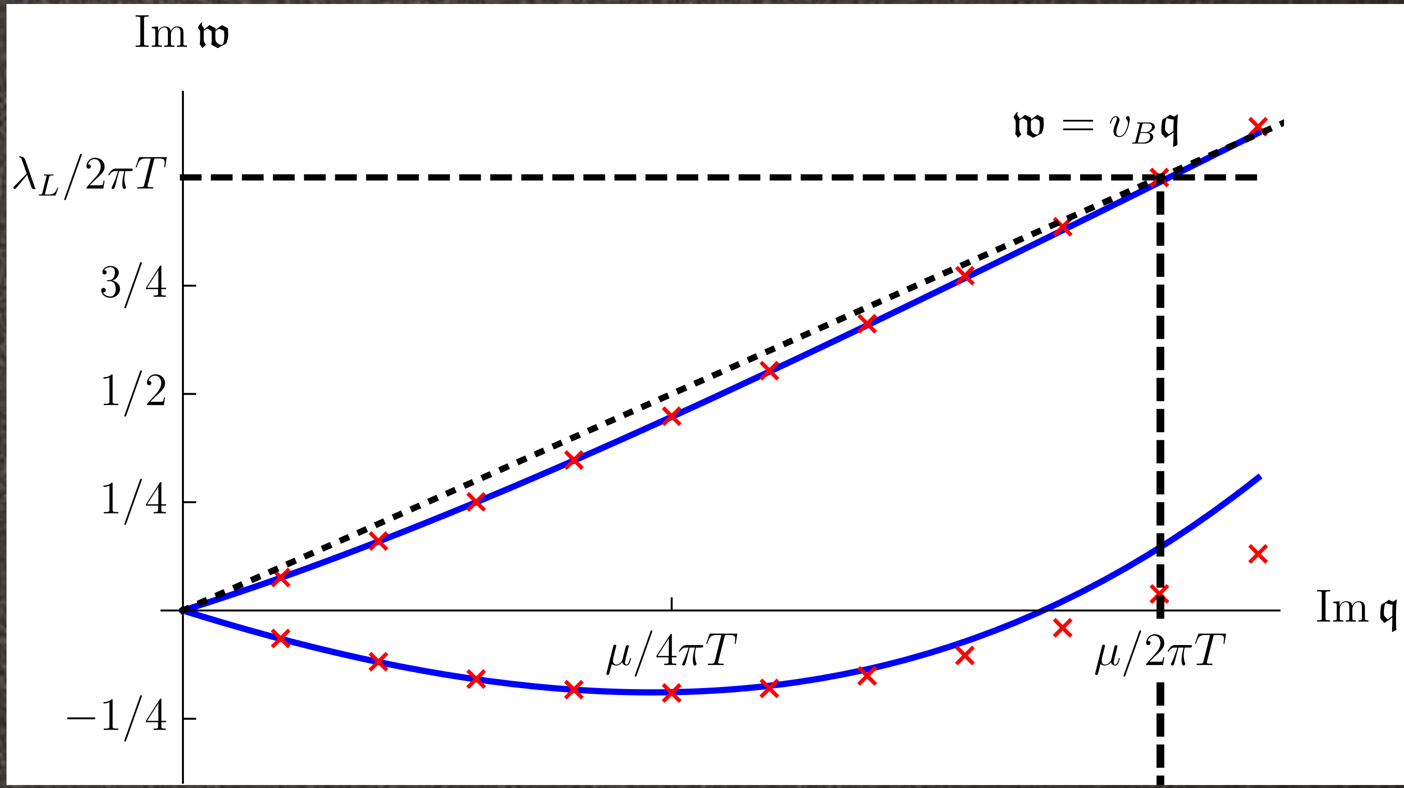
- The information about chaos is recovered from the sound channel spectrum analytically continued to $\mathrm{Im}\,\omega$ and $\mathrm{Im}\,k$

This point is special, since it has <u>vanishing residue</u>

HYDRO SIGNATURE OF CHAOS: POLE-SKIPPING PHENOMENON

$$\mathrm{Res}\, G^R_{T^{00}T^{00}}(\omega = \omega_+^*(i\mu) = i\lambda_L, k = i\mu) = 0$$

Numerically observed in our case



Discussed in Blake, Lee, and Liu (maximal chaos) → Exponential growth implies pole skipping

This solution describing BH scrambling belongs to the spectrum of the sound channel

BH scrambling (quantum chaos), early time and microscopic dynamics

⟵

⟶

Diffusion, late time and collective dynamics

λ_L

η

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Physics should be different between these time scales!

This solution describing BH scrambling belongs to the spectrum of the sound channel

BH scrambling (quantum chaos), early time and microscopic dynamics

Diffusion, late time and collective dynamics

$$\lambda_L \qquad\qquad\qquad \eta$$

Physics should be different between these time scales!

- Another known example of this relation: diluite gas

$$\lambda_L \sim \rho(T)\sqrt{\langle v^2(T)\rangle}\sigma_{2-2}$$

van Zon, van Beijeren, Dellago '98

$$\eta \sim m\frac{\sqrt{\langle v^2(T)\rangle}}{\sigma_{2-2}}$$

Maxwell

BHs satisfy an analogous relation

QUANTUM CHAOS AND HYDRO IN WEAKLY-COUPLED FIELD THEORIES

Based 1804.09182 and work in progress with Grozdanov & Schalm

- We present a new way to derive the kinetic equation from QFT

- In weakly coupled field theories it's possible to recover the OTOC from a retarded Green's function

- This allows to derive a kinetic theory for (quantum) chaos

- Gross number of collisions

BOLTZMANN EQUATION FOR TRANSPORT

- It describes the time evolution of the single-particle distribution function

$$\frac{df(t,\vec{r},\vec{p})}{dt} = \int_{\vec{l}} (R^{gain}(\vec{p},\vec{l}) - R^{loss}(\vec{p},\vec{l}))f(t,\vec{r},\vec{l})$$

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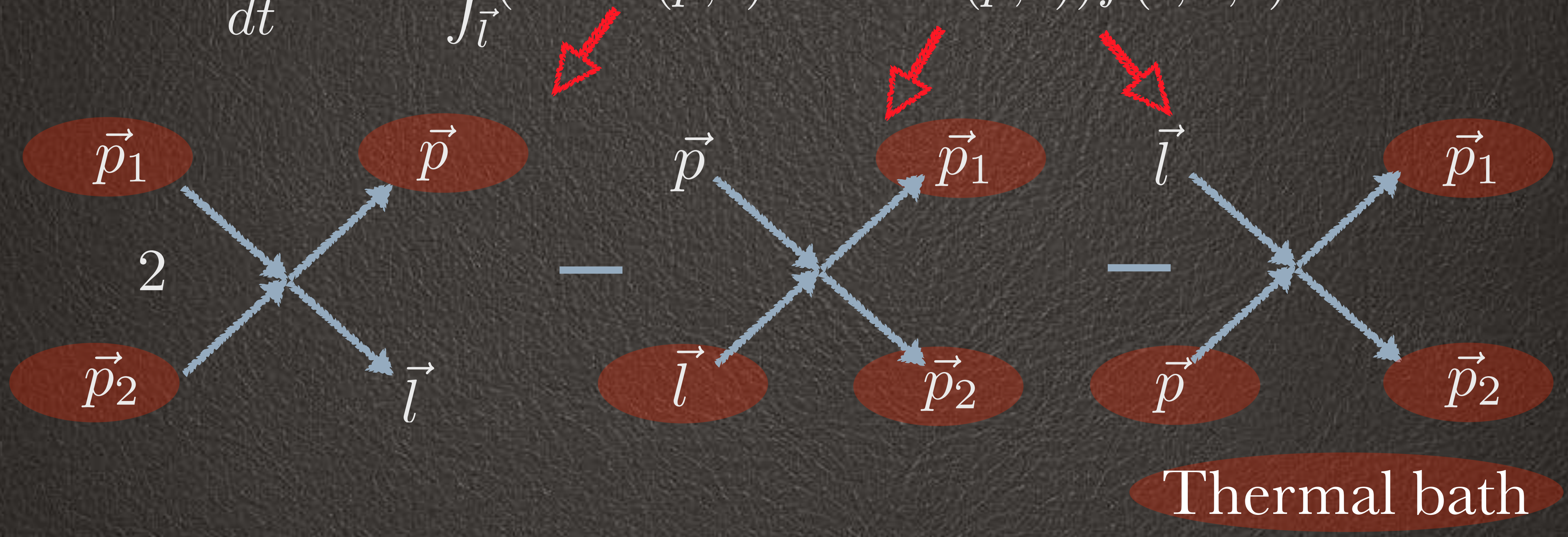

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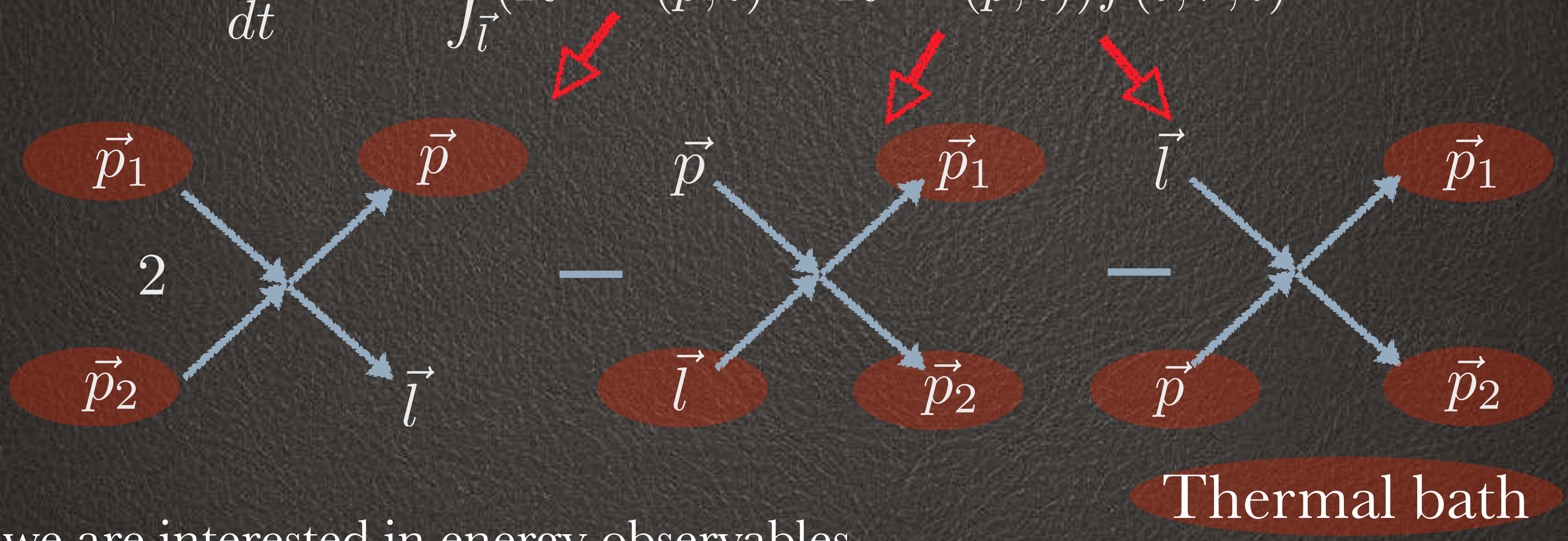

- If we are interested in energy observables

$$f_{En}(t,\mathbf{p}) \equiv \mathcal{E}(E_{\mathbf{p}})f(t,\mathbf{p})$$

$$\frac{df_{En}(t,\vec{r},\vec{p})}{dt} = \int_{\vec{l}} \frac{\mathcal{E}(t,\vec{p})}{\mathcal{E}(t,\vec{l})}(R^{gain}(\vec{p},\vec{l}) - R^{loss}(\vec{p},\vec{l}))f_{En}(t,\vec{r},\vec{l})$$

FROM QFT TO THE KINETIC EQUATION

- Consider the Wigner transform

$$\rho(x,p) = \int_y e^{-ip\cdot y}\,\mathrm{Tr}\left[\Phi(x+y/2)\Phi(x-y/2)\right]$$

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$$\rho(x,\mathbf{p}) = \frac{\delta n(x,\mathbf{p})}{n(\mathbf{p})(1+n(\mathbf{p}))}$$

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$$\rho(x,\mathbf{p}) = \frac{\delta n(x,\mathbf{p})}{n(\mathbf{p})(1+n(\mathbf{p}))}$$

- It satisfies a linearised Boltzmann-like equation

$$\partial_t\rho(t,\mathbf{p}) + \mathcal{L}[\rho](t,\mathbf{p}) = 0$$

FROM QFT TO THE KINETIC EQUATION

$$\partial_t \rho(t, \mathbf{p}) + \mathcal{L}[\rho](t, \mathbf{p}) = 0$$

- The retarded Green's function

$$\begin{aligned} iG_R^{\rho\rho}(x, p|y, q) = &\ \theta(x^0 - y^0)\langle[\rho(x,p), \rho(y,q)]\rangle \\ &= [\partial_{x^0} + \mathcal{L}(x, p|y, q)]^{-1} \ . \end{aligned}$$

- The pole structure of the $G_R^{\rho\rho}(x, p|y, q)$ has all the information regarding the kinetic equation!

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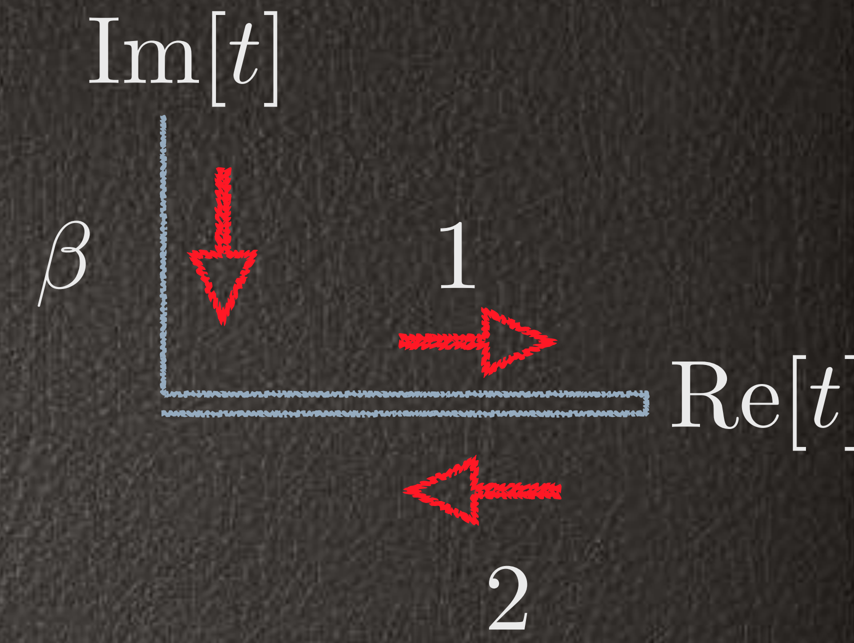
Similarly to $\langle \phi(p)\phi(q) \rangle \propto \dfrac{\delta(p+q)}{p^2 + i\epsilon}$

The Boltzmann equation is relaxational

G_R

HOW DO WE COMPUTE THIS CORRELATOR?



- Schwinger-Keldysh (SK) formalism

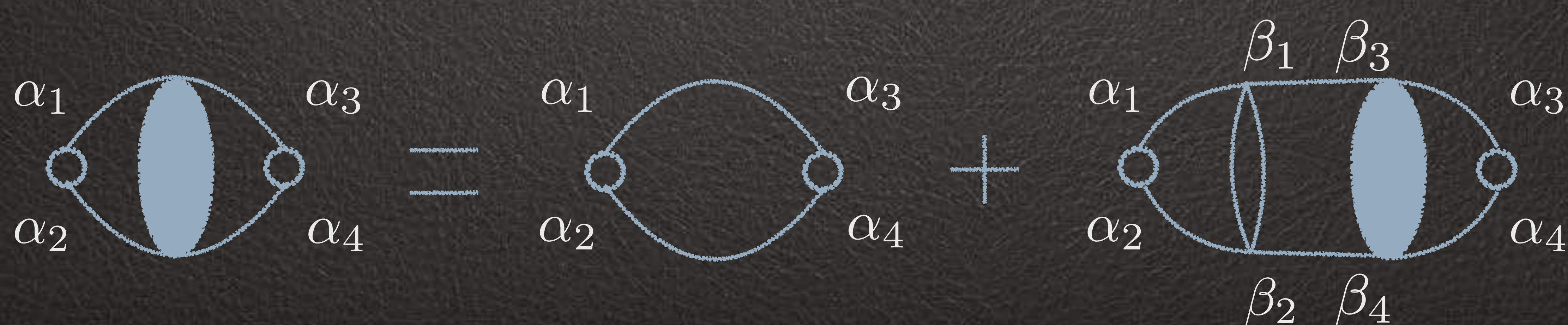
$$\Phi_r = \frac{\Phi_1 + \Phi_2}{2} \qquad \Phi_a = \Phi_1 - \Phi_2$$

SK Branches

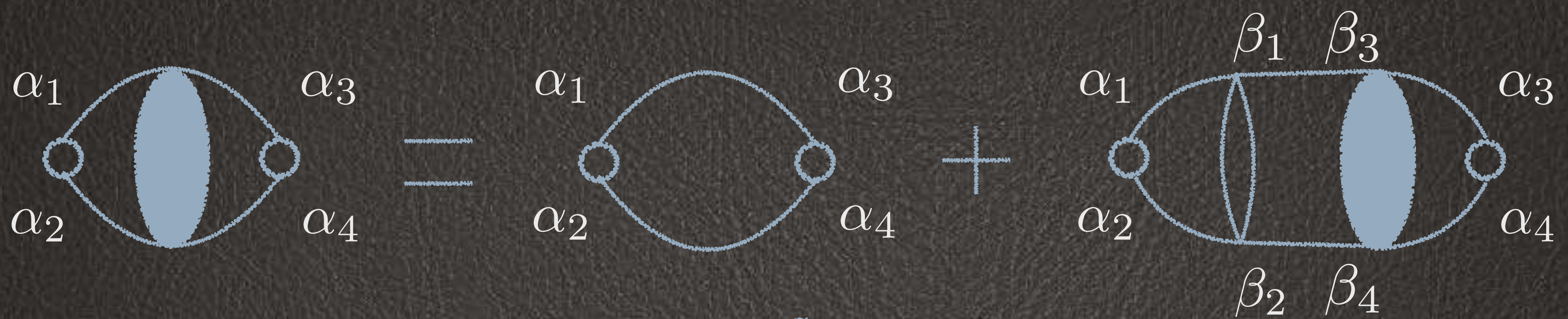
- $G_R^{\rho\rho}$ is a linear combination of 2^4 correlation functions

$$G_{aaar} \quad G_{rara} \quad G_{rraa} \quad \ldots$$

each satisfying a different BSE



- In the hydrodynamic limit $\omega, \mathbf{k} \to 0$



$$G^{\rho\rho}(p|k) = \frac{\pi}{E_{\mathbf{p}}} \frac{\delta(p_0^2 - E_{\mathbf{p}}^2)}{-i\omega + 2\Gamma_{\mathbf{p}}} \left[iN^2 - \int_l \mathcal{K}(p,l) G^{\rho\rho}(l|k) \right].$$

Imaginary part of self-energy

Yaffe, Jeon '95

Wang and Heinz '03

$$G^{\rho\rho}(p|k) = \frac{\pi}{E_{\mathbf{p}}} \frac{\delta(p_0^2 - E_{\mathbf{p}}^2)}{-i\omega + 2\Gamma_{\mathbf{p}}} \left[iN^2 - \int_l \mathcal{K}(p,l) G^{\rho\rho}(l|k) \right].$$

- The solution is supported on-shell

$$G^{\rho\rho}(p|k) = \delta(p_0^2 - E_{\mathbf{p}}^2) G^{ff}(\mathbf{p}|k)$$

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- So the BSE becomes

$$-i\omega G^{ff}(\mathbf{p}|k) = \frac{i\pi N^2}{E_{\mathbf{p}}} - 2\Gamma_{\mathbf{p}} G^{ff}(\mathbf{p}|k) - \int_{\mathbf{l}} \frac{\mathcal{K}(E_{\mathbf{p}} + E_{\mathbf{l}}) + \mathcal{K}(E_{\mathbf{p}} - E_{\mathbf{l}})}{2E_{\mathbf{p}}} G^{ff}(\mathbf{l}|k)$$

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R^{loss} R^{gain}

- These terms are <u>exactly</u> the R^{loss} and R^{gain}

$$G^{ff}(\mathbf{p}|k) = \frac{1}{-i\omega - \int_{\mathbf{l}}(R^{gain}(\mathbf{p},\mathbf{l}) - R^{loss}(\mathbf{p},\mathbf{l}))}\,\frac{i\pi N^2}{E_{\mathbf{p}}}$$

HOW IS THIS RELATED TO THE OTOC?

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- There exist another ansatz to the previous BSE

$$\mathcal{E}^{-1}(E)$$

$$G^{*}_{aarr}(p|k) = \delta(p_0^2 - E_{\mathbf{p}}^2)\, sinh(\beta p_0/2)\, G^{ff}(\mathbf{p}|k)$$

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- This is exactly the BSE for the OTOC!

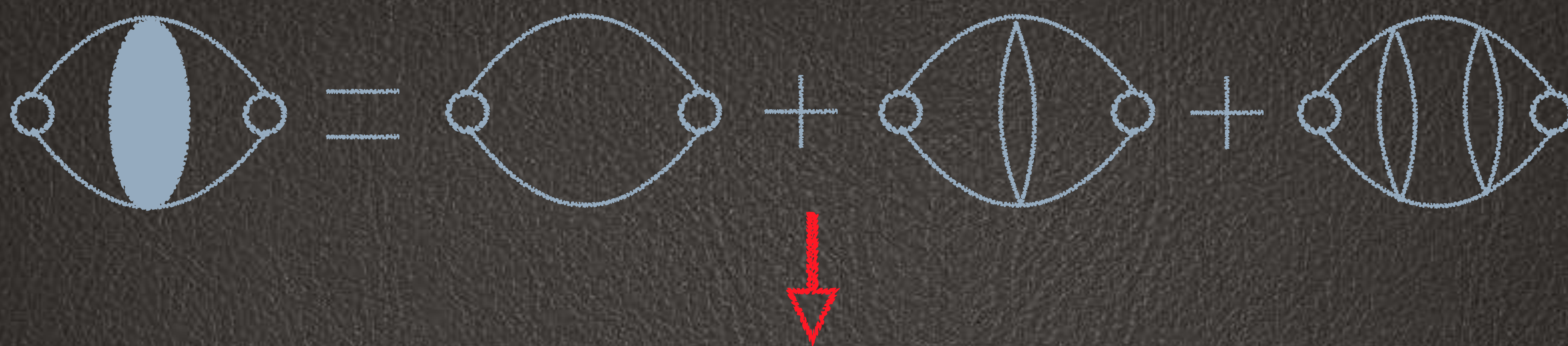
Stanford '15

$$(-i\omega + 2\Gamma_{\mathbf{p}}) G^{\text{ff}}(\mathbf{p}|k) = \frac{i\pi N^2}{E_{\mathbf{p}}} - \int_{\mathbf{l}} \frac{\sinh(\beta E_{\mathbf{l}}/2)}{\sinh(\beta E_{\mathbf{p}}/2)} \frac{\mathcal{K}(E_{\mathbf{p}} - E_{\mathbf{l}}) - \mathcal{K}(E_{\mathbf{p}} + E_{\mathbf{l}})}{2E_{\mathbf{p}}} G^{\text{ff}}(\mathbf{l}|k)\,.$$

Computing the Ljapunov exponent

Stanford '15

Patel, Chowdhury, Sachte and Swingle '17



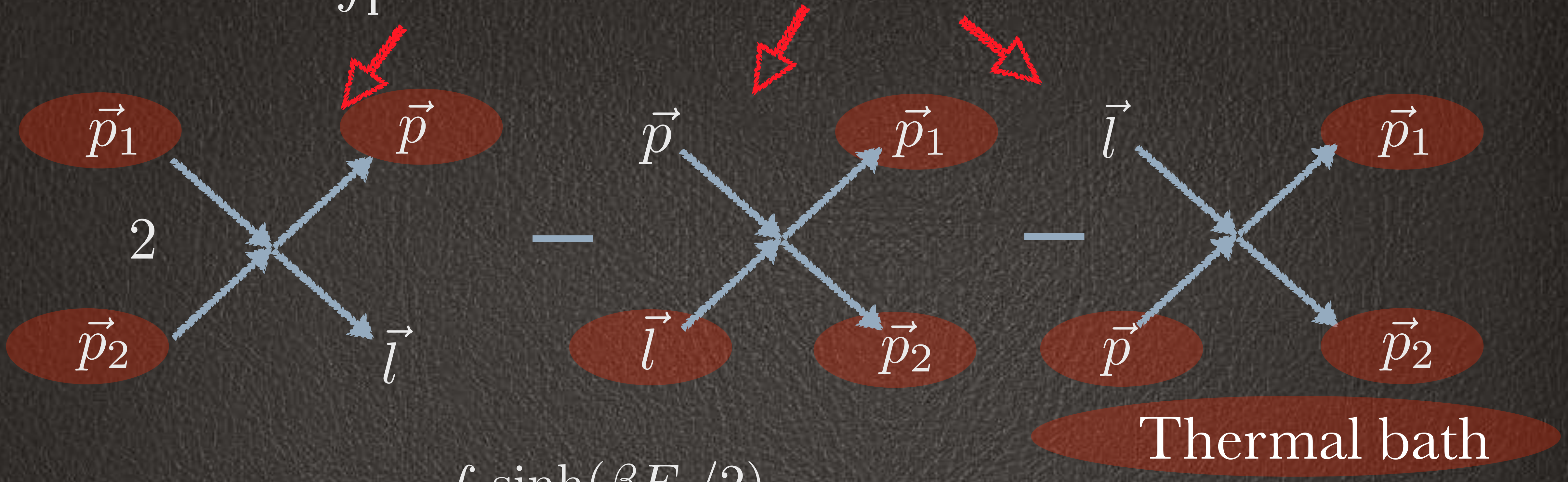
$$\partial_t G(t|\mathbf{p}) = \mathrm{I} + \hat{M}(\mathbf{p},\mathbf{l})G(t|\mathbf{l})$$

- The exponential behaviour is determined by the homogeneous part

- The Lyapunov exponents are related to the eigenvalues of the operator

$$\lambda_L G(t|\mathbf{p}) = \partial_t G(t|\mathbf{p}) = \hat{M}(\mathbf{p},\mathbf{l})G(t|\mathbf{l})$$

$$\partial_t f(t,\mathbf{p}) = \int_{\mathbf{l}} \left[R^{gain}(\mathbf{p},\mathbf{l}) - R^{loss}(\mathbf{p},\mathbf{l})\right] f(t,\mathbf{l})\,.$$



$$\partial_t f_{OTOC}(t,\mathbf{p}) = \int_{\mathbf{l}} \frac{\sinh(\beta E_{\mathbf{l}}/2)}{\sinh(\beta E_{\mathbf{p}}/2)}$$

$$\times \left[R^{gain}(\mathbf{p},\mathbf{l}) + R^{loss}(\mathbf{p},\mathbf{l}) - 4\Gamma_{\mathbf{p}}\delta(\mathbf{p}-\mathbf{l})\right] f_{OTOC}(t,\mathbf{l})\,.$$

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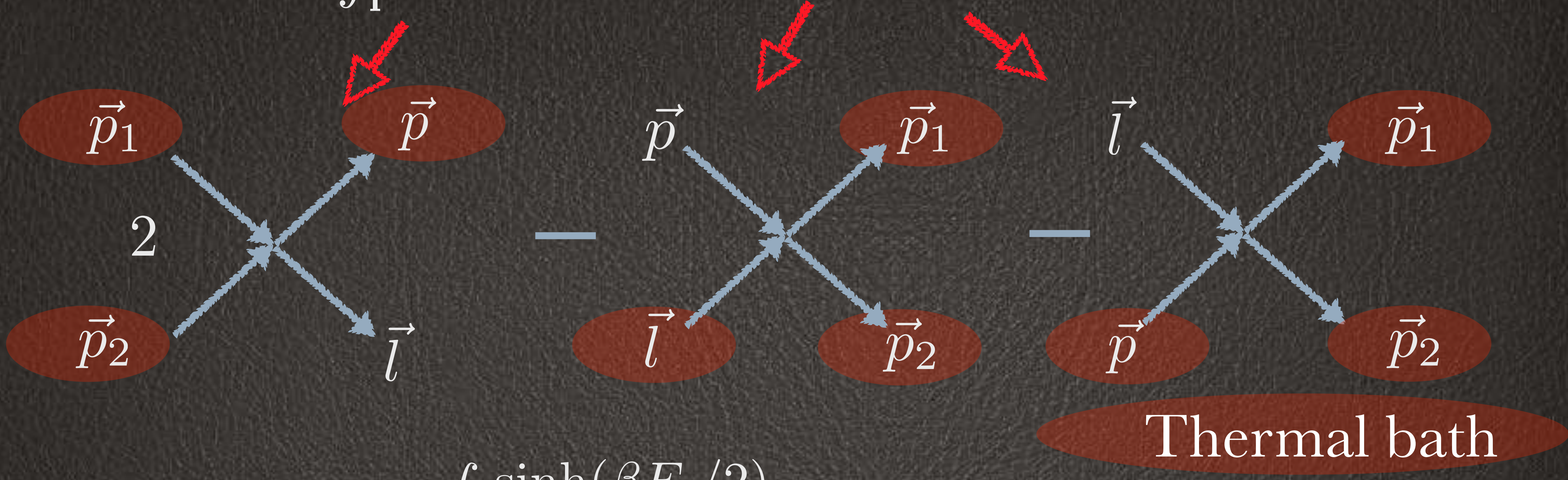
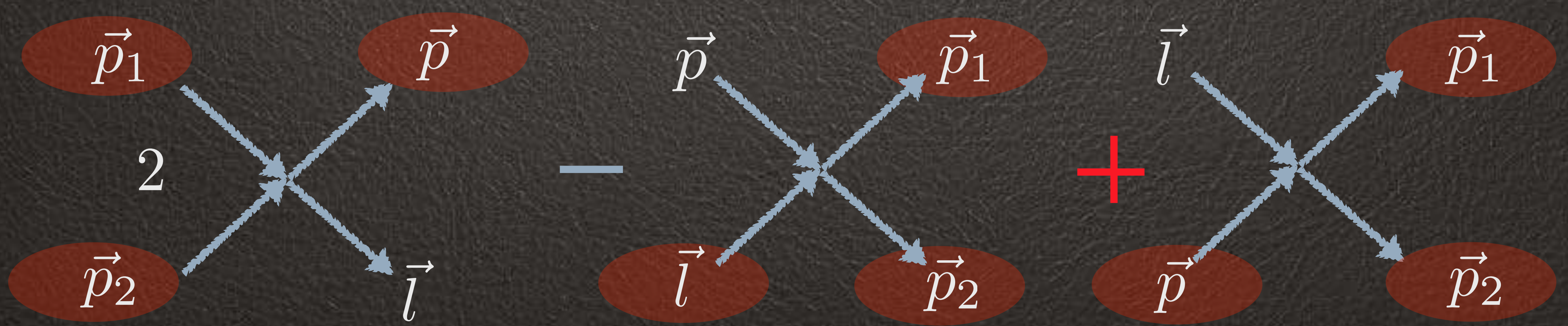


$$\partial_t f_{OTOC}(t,\mathbf{p}) = \int_{\mathbf{l}} \frac{\sinh(\beta E_{\mathbf{l}}/2)}{\sinh(\beta E_{\mathbf{p}}/2)}$$

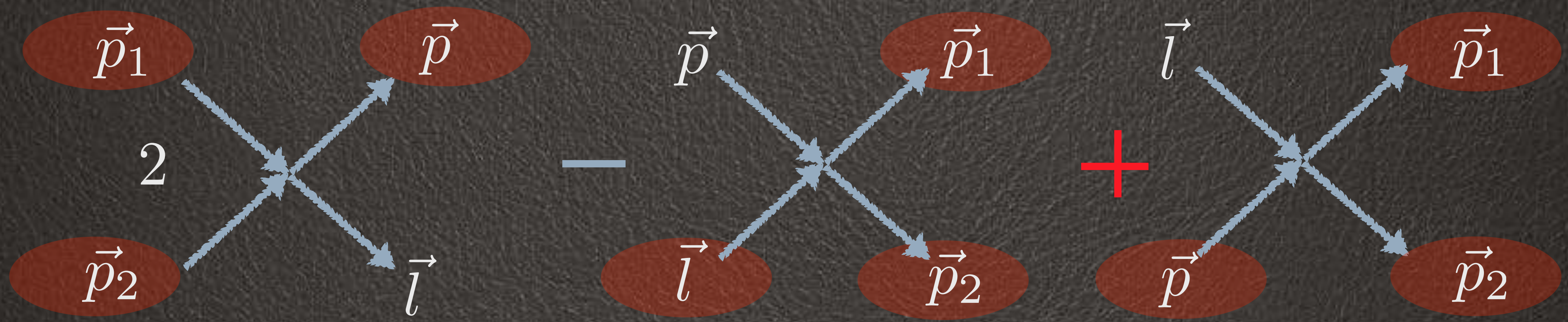
$$\times \left[R^{gain}(\mathbf{p},\mathbf{l}) + R^{loss}(\mathbf{p},\mathbf{l}) - 4\Gamma_{\mathbf{p}}\delta(\mathbf{p}-\mathbf{l})\right] f_{OTOC}(t,\mathbf{l})\,.$$



Moreover

The kernel is mainly supported $\mathbf{p} \approx \mathbf{l}$

$$\partial_t f_{OTOC}(t,\mathbf{p}) = \int_{\mathbf{l}} \left[R^{gain}(\mathbf{p},\mathbf{l}) + R^{loss}(\mathbf{p},\mathbf{l}) - 4\Gamma_{\mathbf{p}}\delta(\mathbf{p}-\mathbf{l}) \right] f_{OTOC}(t,\mathbf{l})\,.$$



Gross number of collision

FROM THE BSE TO KINETIC EQUATION

- Transport: negative definite spectrum

$$\partial_t f(t,\mathbf{p}) = \int_{\mathbf{l}} \left[R^{gain}(\mathbf{p},\mathbf{l}) - R^{loss}(\mathbf{p},\mathbf{l})\right] f(t,\mathbf{l})\,.$$

$$f(t,\mathbf{p}) \propto e^{\lambda t} \quad \lambda \leq 0$$

- OTOC: non-negative definite spectrum

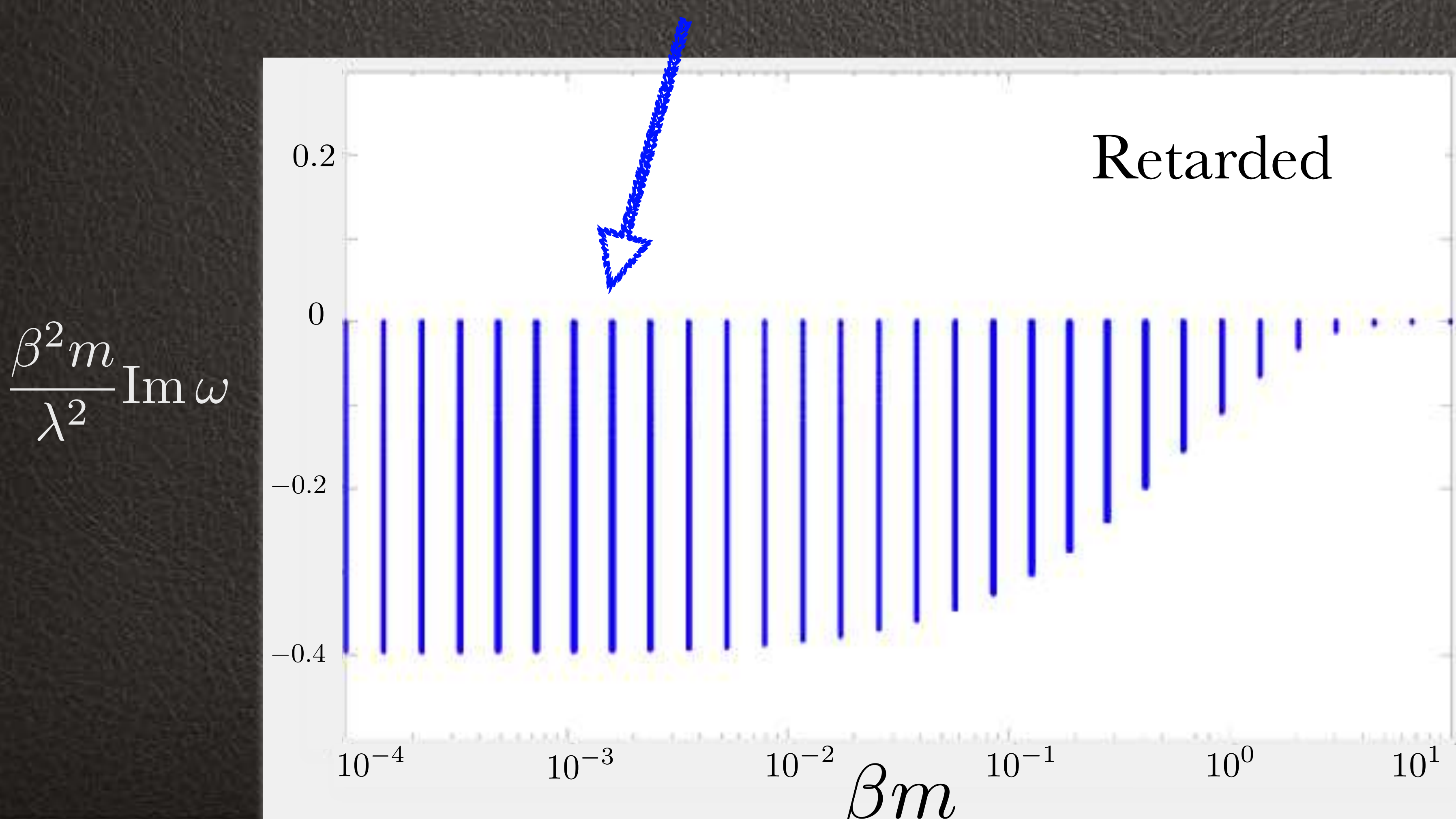
$$\partial_t f_{OTOC}(t,\mathbf{p}) = \int_{\mathbf{l}} \left[R^{gain}(\mathbf{p},\mathbf{l}) + R^{loss}(\mathbf{p},\mathbf{l}) - 4\Gamma_{\mathbf{p}}\delta(\mathbf{p}-\mathbf{l})\right] f_{OTOC}(t,\mathbf{l})\,.$$

$$f^{OTOC}(t,\mathbf{p}) \propto e^{\lambda t} \quad \lambda \not\leq 0$$

Front propagating into unstable states

Van Saarloos (review on front)

Poles (BC?)



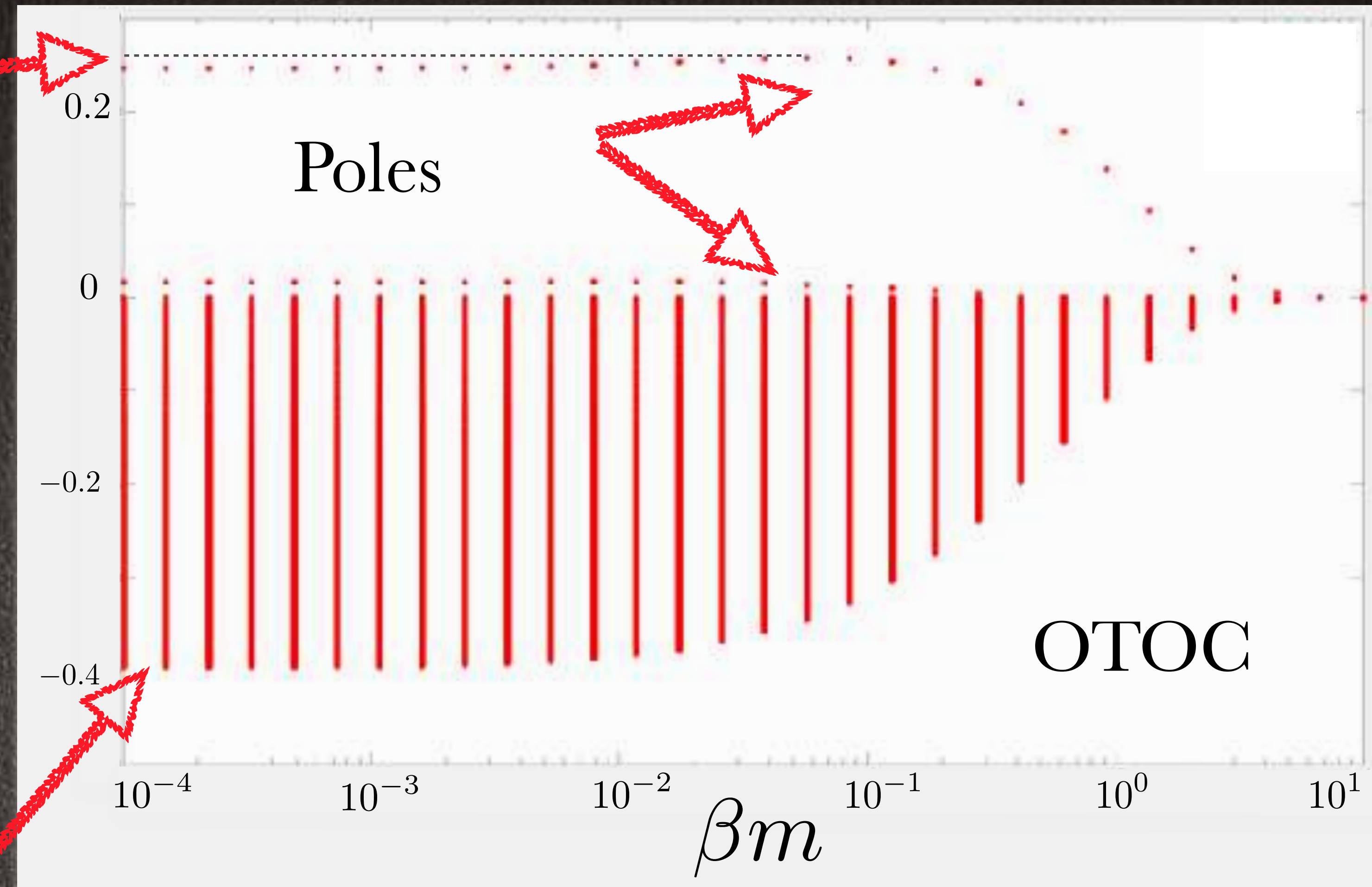
Ret. spectrum

Stanford

OTOC spectrum

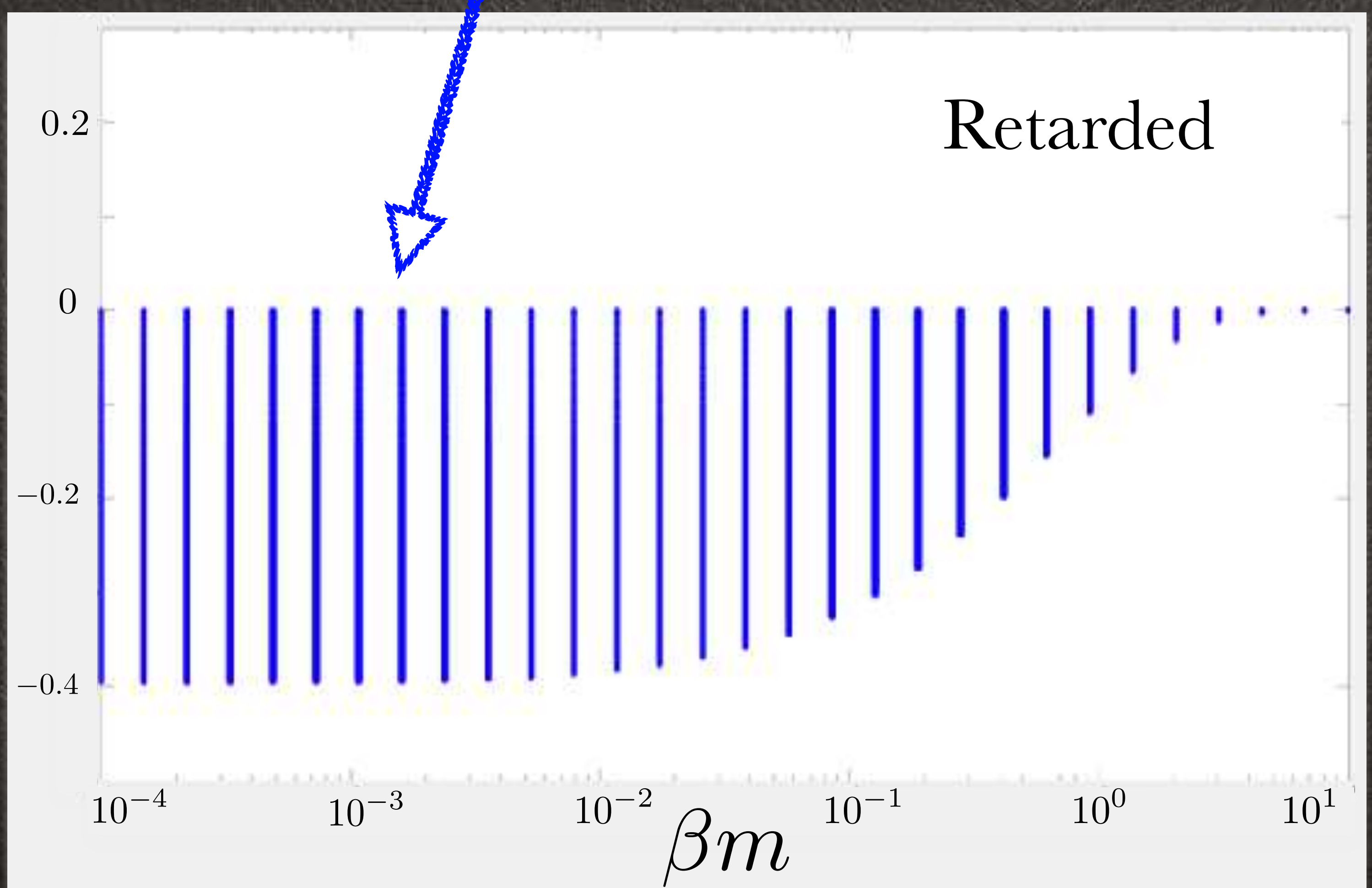
$\frac{\beta^2 m}{\lambda^2}\lambda_L$

$\frac{\beta^2 m}{\lambda^2}\mathrm{Im}\,\omega$



Poles (BC?)

$\frac{\beta^2 m}{\lambda^2}\mathrm{Im}\,\omega$



Ret. spectrum

Moore ‘18

ANALOGIES WITH THE CLASSICAL GAS

- Gross-energy exchange

$$\partial_t f_{OTOC}(t, \mathbf{p}) = \int_{\mathbf{l}} \left[R^{gain}(\mathbf{p}, \mathbf{l}) + R^{loss}(\mathbf{p}, \mathbf{l}) - 4\Gamma_{\mathbf{p}} \delta(\mathbf{p} - \mathbf{l}) \right] f_{OTOC}(t, \mathbf{l}) \,.$$

- Clock model for hard spheres and hard disks (ad-hoc)

$$\partial_t f_k = -f_k + f_{k-1}^2 + 2 f_{k-1} \sum_{l=0}^{k-2} f_l$$

van Zon, van Beijeren, Dellago '98

Density of particles with clock value k

Front propagating into an unstable state

Connections with transport

$$\lim_{\omega, k_z \to 0} G_R^{T^{xy}T^{xy}}(\omega, k_z) = \lim_{\omega, k_z \to 0} \beta\omega \int_p p_x p_y G_R^{\rho\rho}(p|\omega, k_z)$$

- The shear viscosity is related to this Green's function by the Kubo formula

$$\eta = \lim_{\omega, k_z \to 0} \frac{1}{i\omega} \mathrm{Im} G_R^{\Pi^{xy}\Pi^{xy}}(\omega, k_z)$$

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$$\eta = \lim_{\omega,k_z\to 0} \frac{1}{i\omega}\mathrm{Im}G_R^{\Pi^{xy}\Pi^{xy}}(\omega,k_z)$$

- The OTOC corresponds to an analytic continuation of $G_R^{\rho\rho}$ which is odd in p_0

$$\int_p p_x p_y G_R^{(OTOC)} = 0$$

WHAT IS THE ROLE OF LARGE N?

The BSE is formally equivalent to

$$\left(\frac{d}{dt} - g^4 N^2 L\right) f = N^2$$

Whose solution

$$f = -\frac{1}{g^4 L} + c_0\, e^{g^4 N^2 L t}$$

The scrambling time

$$t_{scr} = \frac{1}{g^4 N^2 L} \ln(1/g^4 L c_0)$$

There is no need for large N

CONCLUSIONS

In holographic theories

- Scrambling is related to hydrodynamics: analytical continuation of a sound pole

Pole-skipping → No need for OTOC

In weakly coupled field theories

- The kinetic equation can be derived from QFT by the analytical structure of an correlation function
- OTOC emerges as a analytical continuation of this correlation function
- This allows to derive the kinetic equation for quantum chaos

$$\partial_t f_{OTOC}(t,\mathbf{p}) = \int_{\mathbf{l}} \left[R^{gain}(\mathbf{p},\mathbf{l}) + R^{loss}(\mathbf{p},\mathbf{l}) - 4\Gamma_{\mathbf{p}}\delta(\mathbf{p}-\mathbf{l})\right] f_{OTOC}(t,\mathbf{l})\,.$$

THANKS