Quantum phases of SYK models

Subir Sachdev
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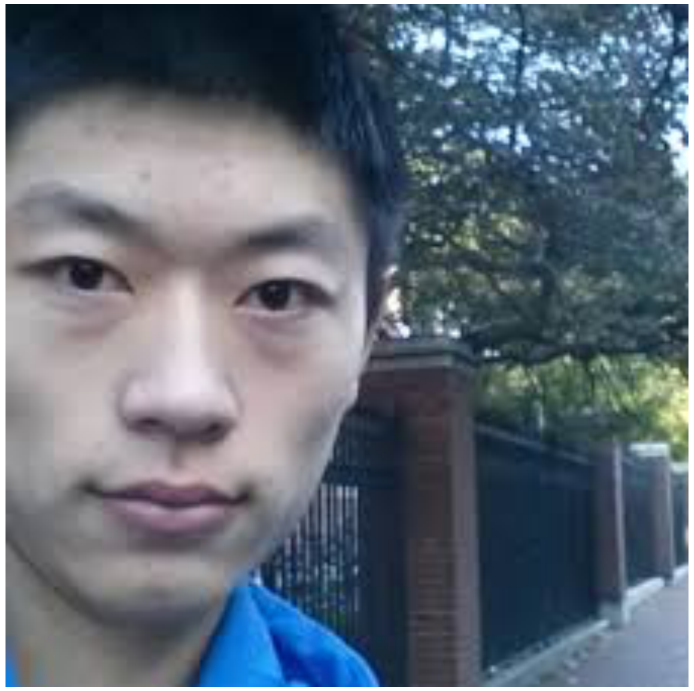
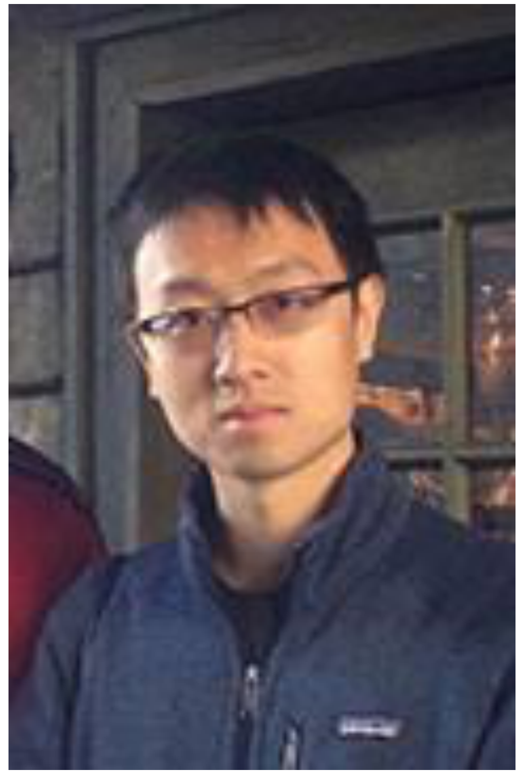
Integrable and Chaotic Quantum Dynamics:
from Holography to Lattice,
Bled, Slovenia

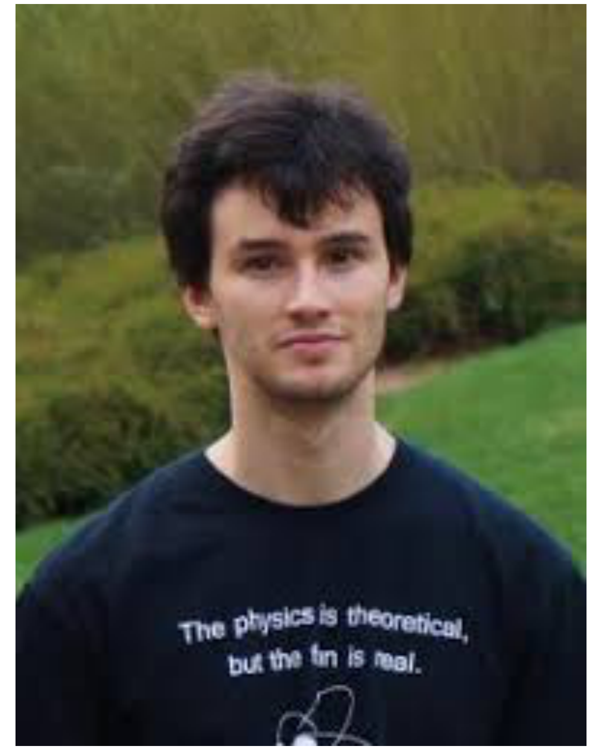
PHYSICS
HARVARD



Wenbo Fu
Harvard



Yingfei Gu
Harvard



Grisha Tarnopolsky
Harvard

arXiv:1804.04130

Aavishkar Patel

To appear

Quantum matter <u>with</u> quasiparticles:

The quasiparticle idea is the key reason for the many successes of quantum condensed matter physics:

- Fermi liquid theory of metals, insulators, semiconductors
- Theory of superconductivity (pairing of quasiparticles)
- Theory of disordered metals and insulators (diffusion and localization of quasiparticles)
- Theory of metals in one dimension (collective modes as quasiparticles)
- Theory of the fractional quantum Hall effect (quasiparticles which are `fractions' of an electron)

Quantum matter with quasiparticles:

- **Quasiparticles are additive excitations:**
  The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

In a lattice system of N sites, this parameterizes the energy of $\sim e^{\alpha N}$ states in terms of poly(N) numbers.

Quantum matter with quasiparticles:

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\rm eq} \sim \frac{\hbar E_F}{(k_B T)^2} \quad , \quad \text{as } T \to 0,$$

where E_F is the Fermi energy.

1. Metal with quasiparticles
   Random matrix model of a `quantum dot'

2. Metal without quasiparticles
   SYK model of a `quantum dot'

3. Lattice models of SYK islands
   Theory of a strange metal

4. Z_2 Fractionalization in a SYK *t-J* model

5. SYK U(1) gauge theory

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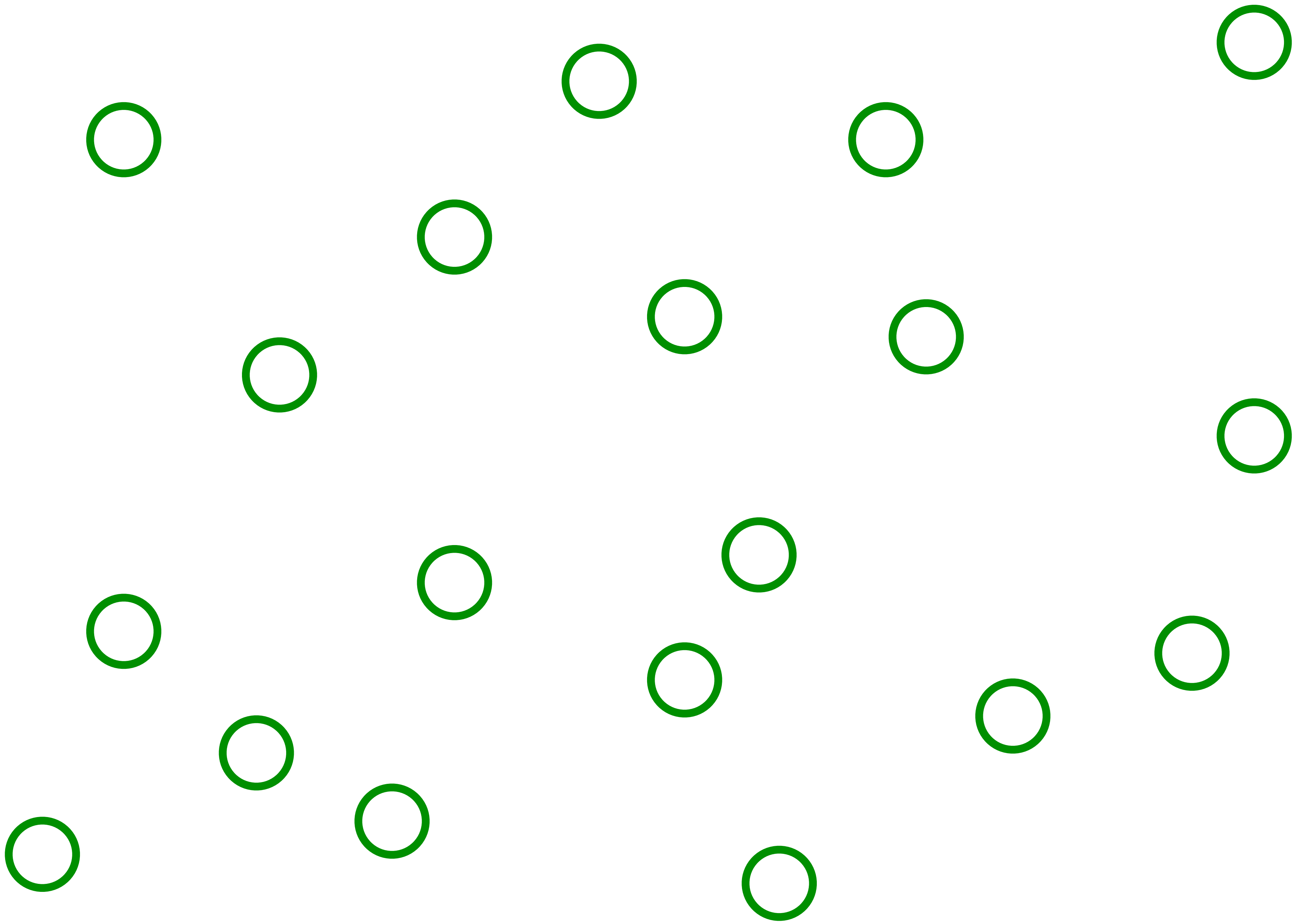
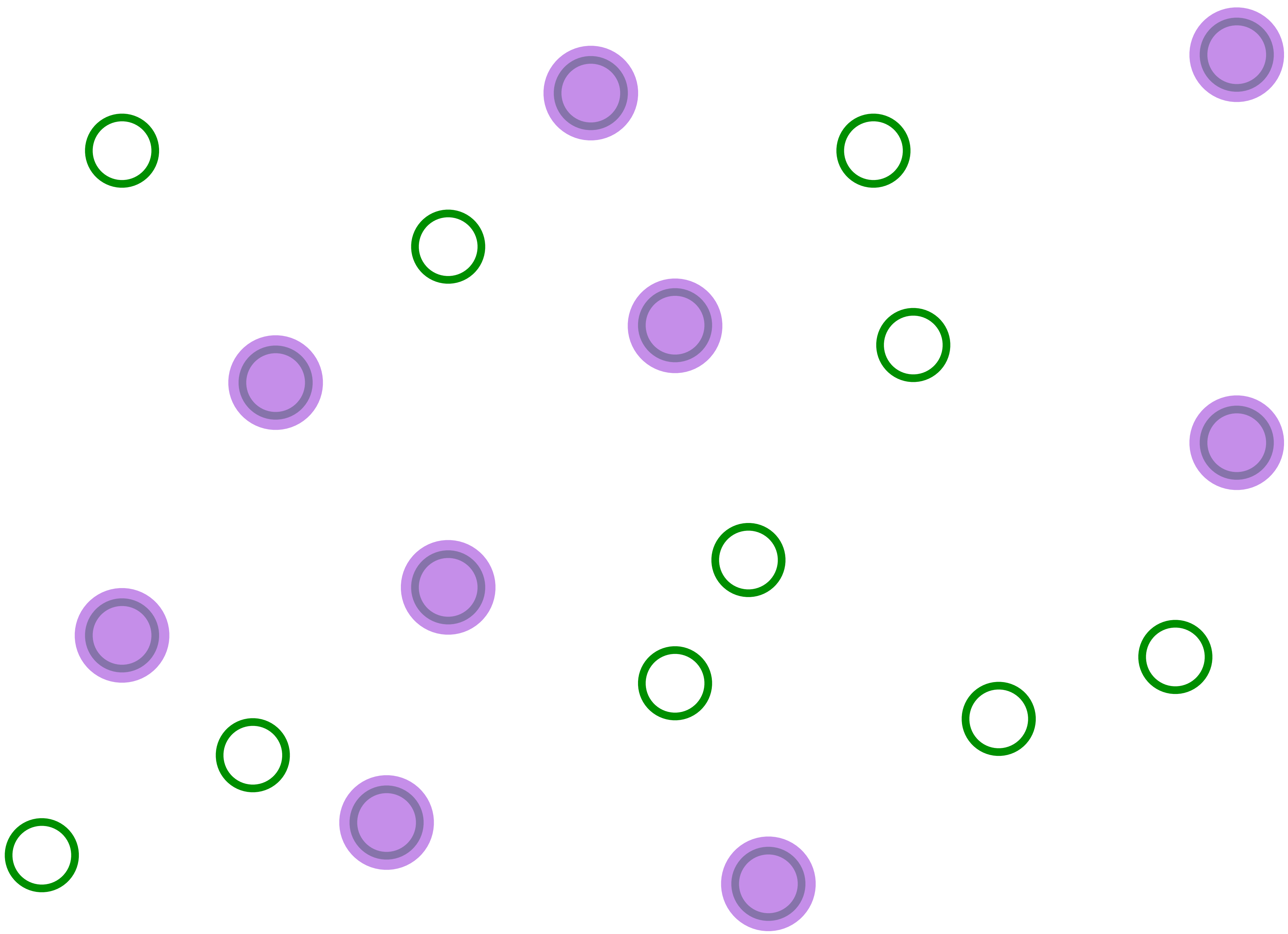


Pick a set of random positions

A simple model of a metal with quasiparticles



Place electrons randomly on some sites

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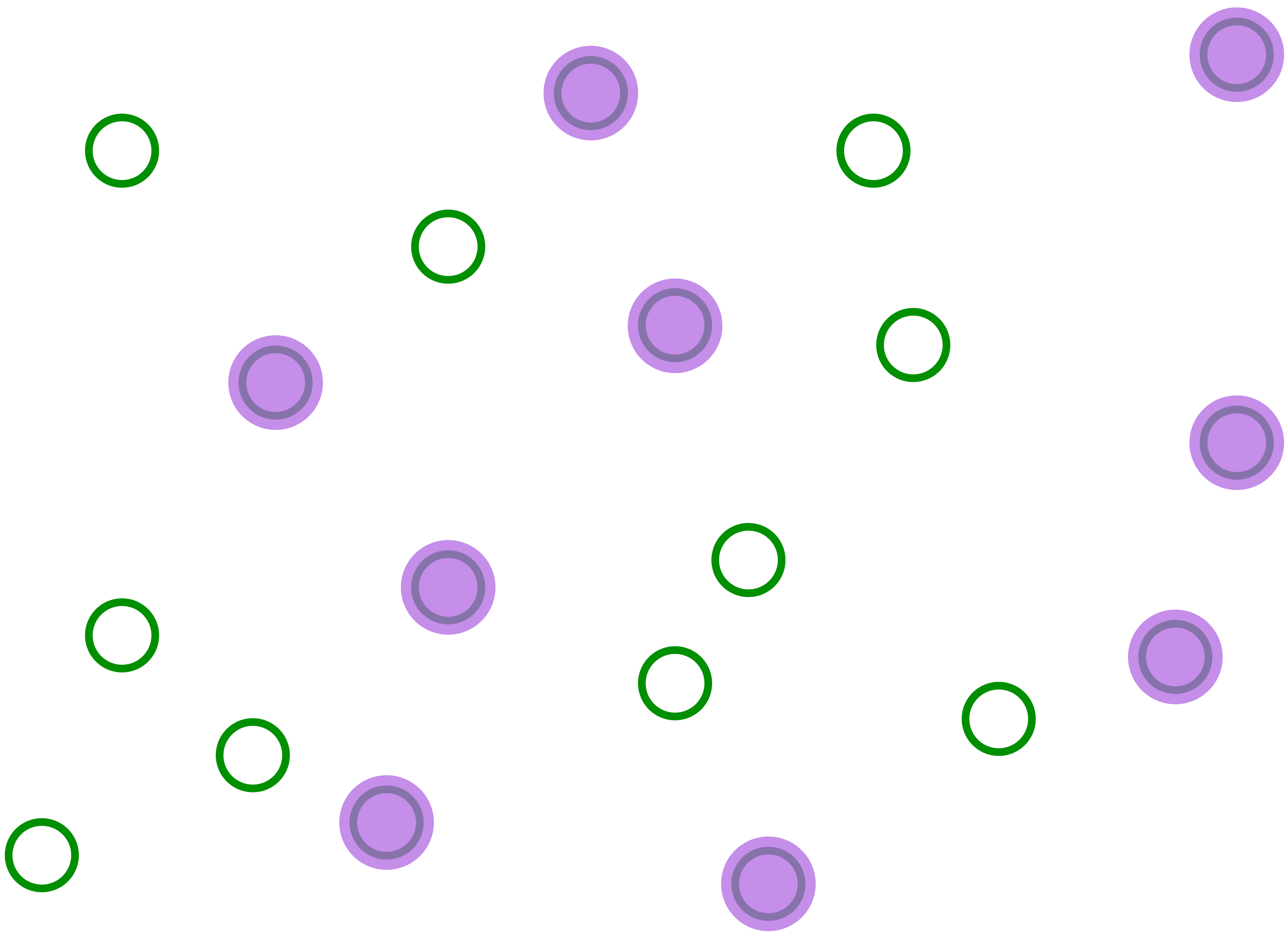

Electrons move one-by-one randomly

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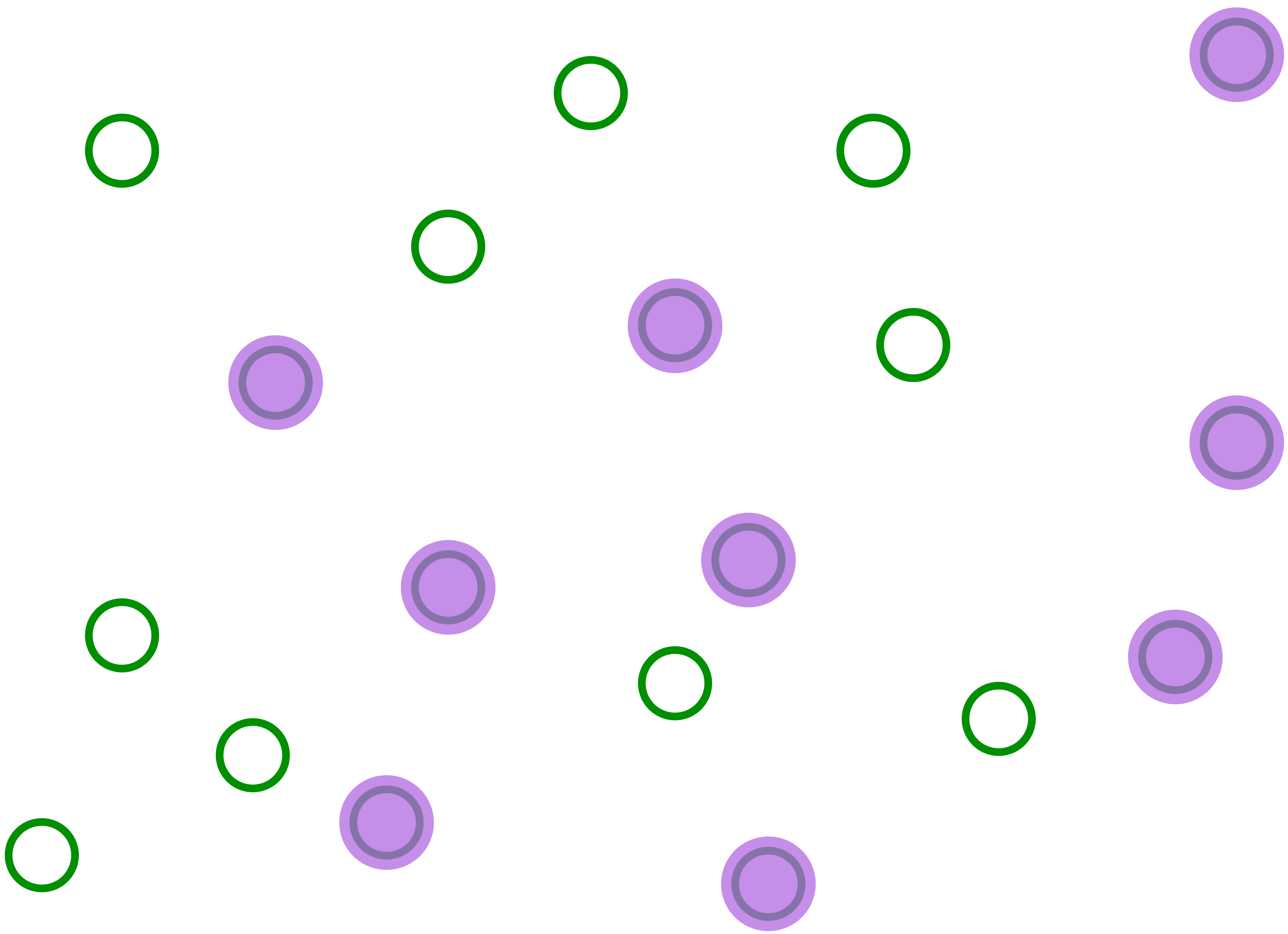

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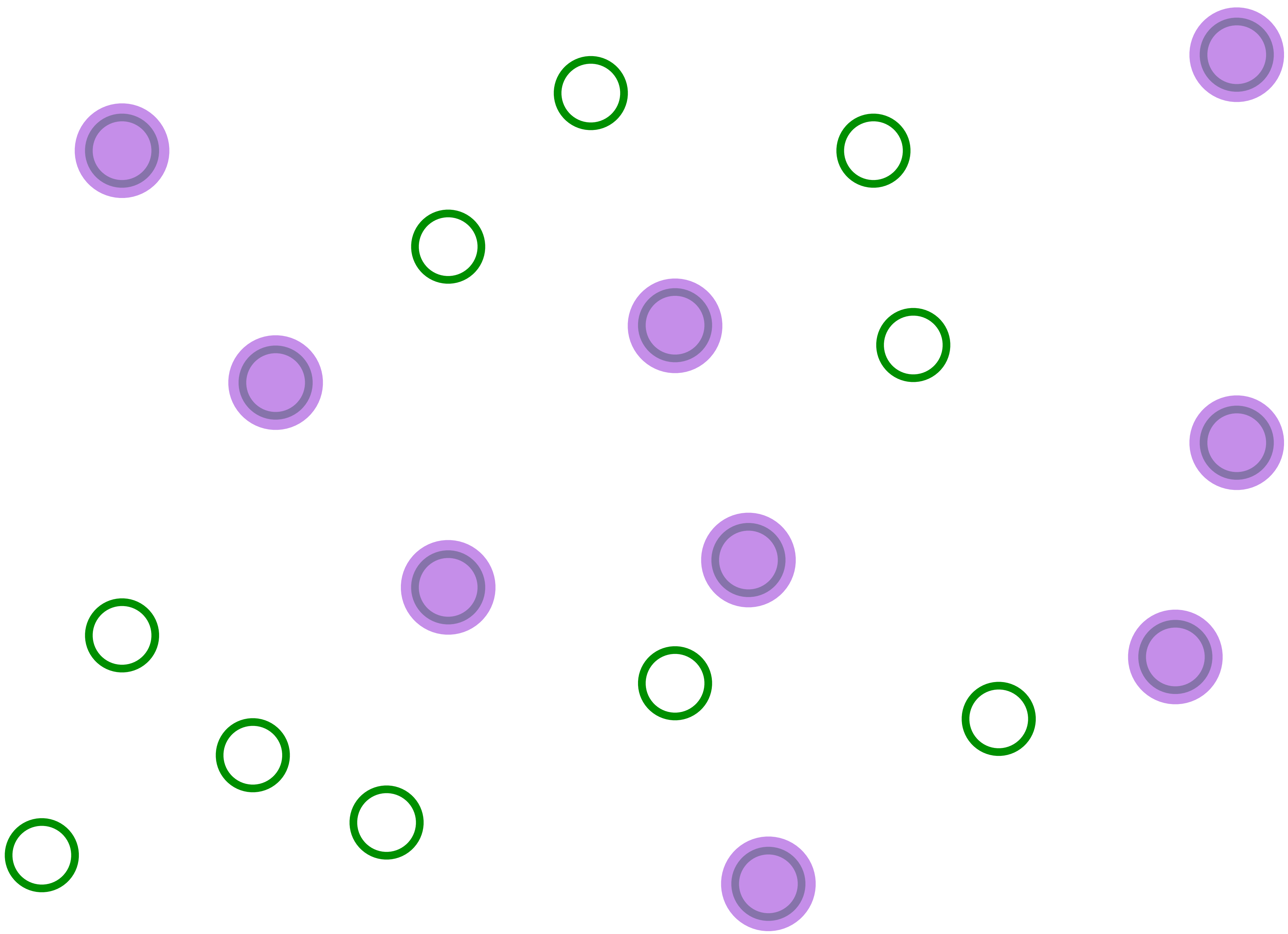

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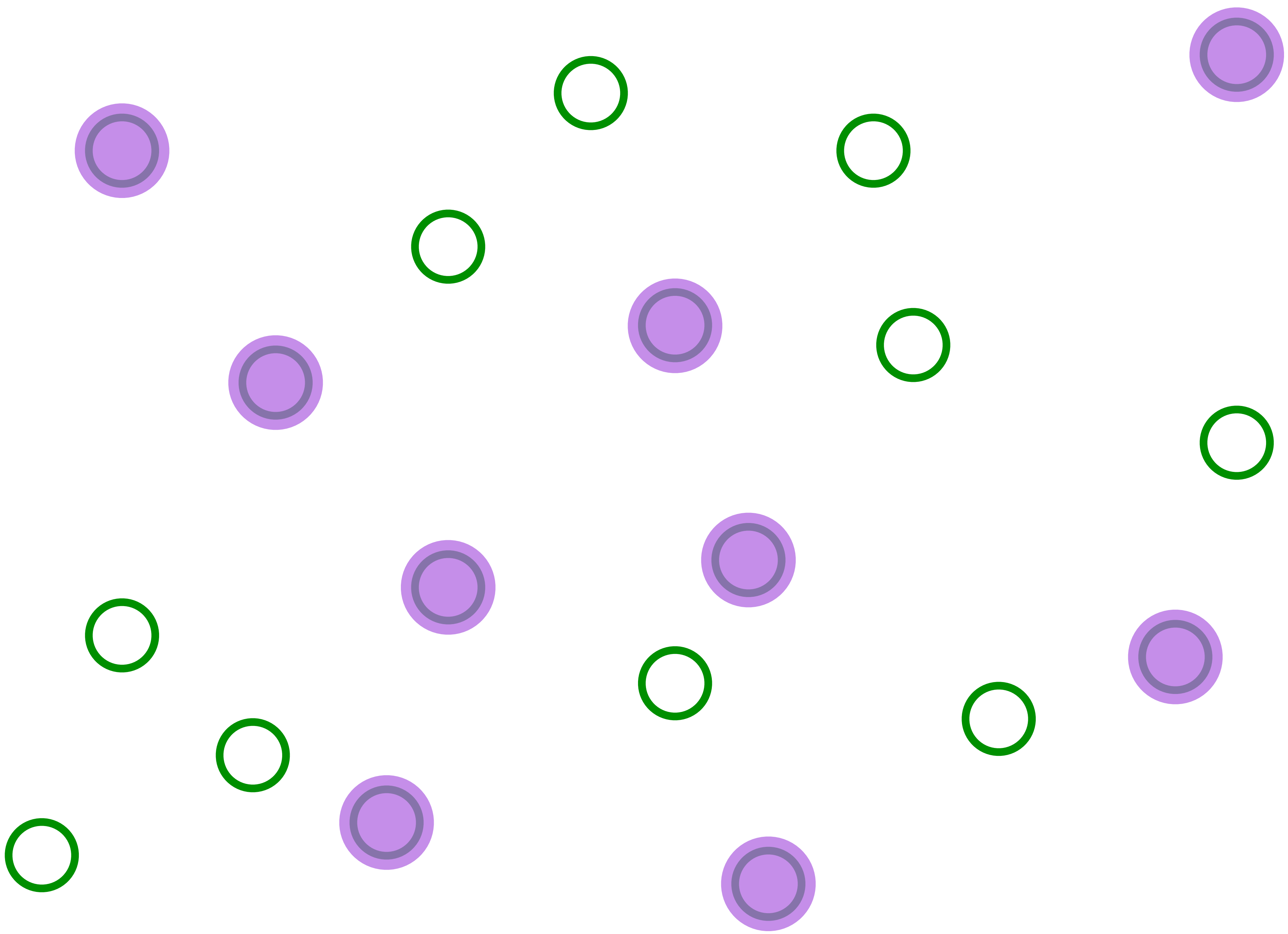

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A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

Fermions occupying the eigenstates of a *N* x *N* random matrix

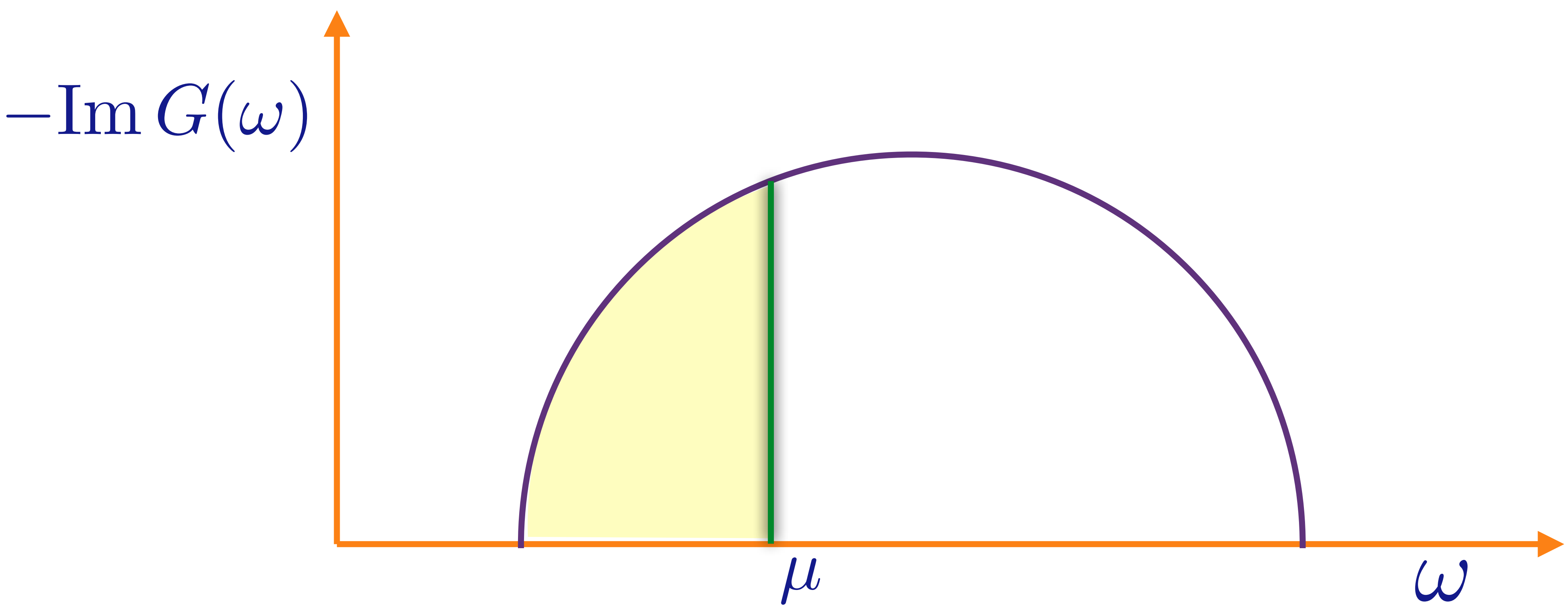
Infinite-range model with quasiparticles

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

$G(\omega)$ can be determined by solving a quadratic equation.



Infinite-range model with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

$J_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$. We compute the lifetime of a quasiparticle, τ_α, in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamitonian with energy ε_α. By Fermi's Golden rule, for ε_α at the Fermi energy

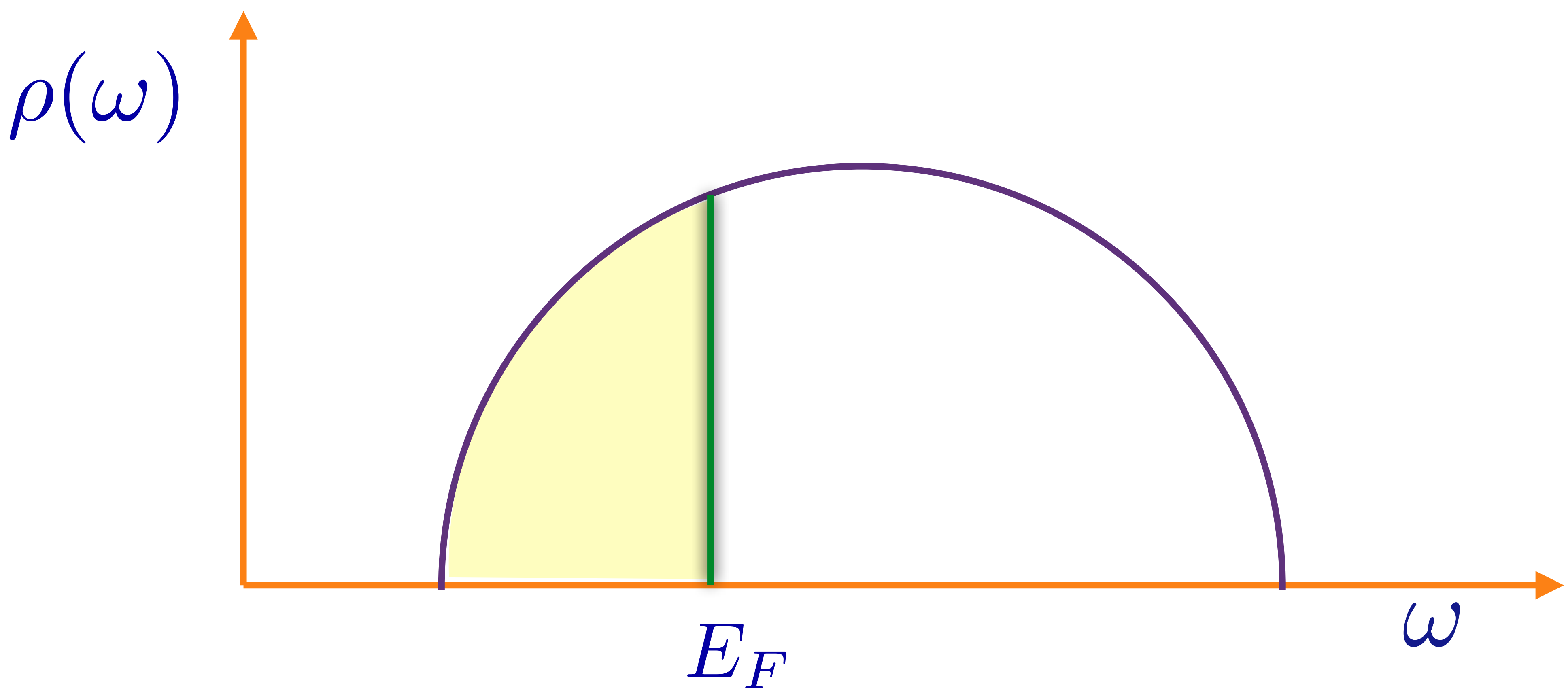
$$\begin{aligned} \frac{1}{\tau_\alpha} &= \pi U^2 \rho_0^2 \int d\varepsilon_\beta d\varepsilon_\gamma d\varepsilon_\delta f(\varepsilon_\beta)(1 - f(\varepsilon_\gamma))(1 - f(\varepsilon_\delta))\delta(\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_\gamma - \varepsilon_\delta) \\ &= \frac{\pi^3 U^2 \rho_0^2}{4} T^2 \end{aligned}$$

where ρ_0 is the density of states at the Fermi energy.

Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.

A simple model of a metal with quasiparticles

Let ε_α be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F. The density of states is $\rho(\omega) = (1/N)\sum_\alpha \delta(\omega - \varepsilon_\alpha)$.



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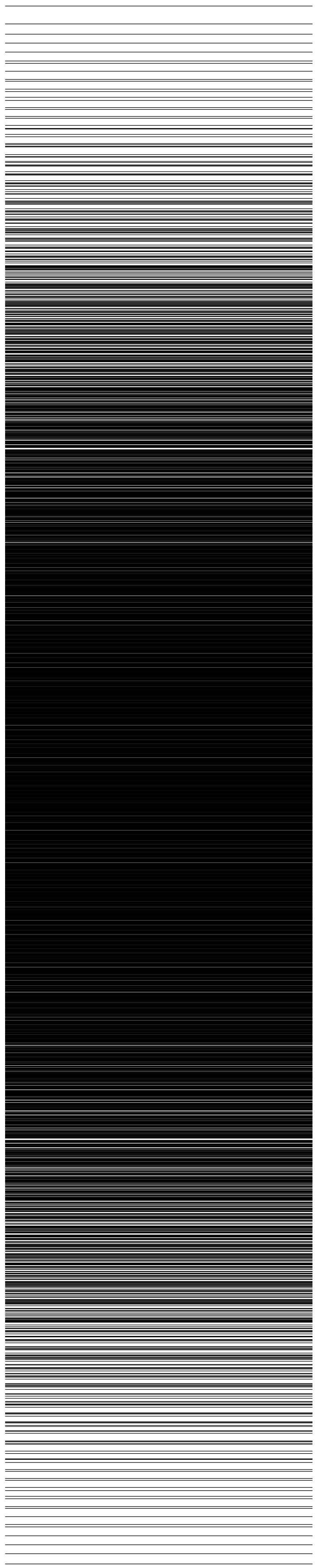

Many-body level spacing $\sim 2^{-N}$

Quasiparticle excitations with spacing $\sim 1/N$

There are 2^N many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha,$$

where $n_\alpha = 0, 1$. Shown are all values of E for a single cluster of size $N = 12$. The ε_α have a level spacing $\sim 1/N$.

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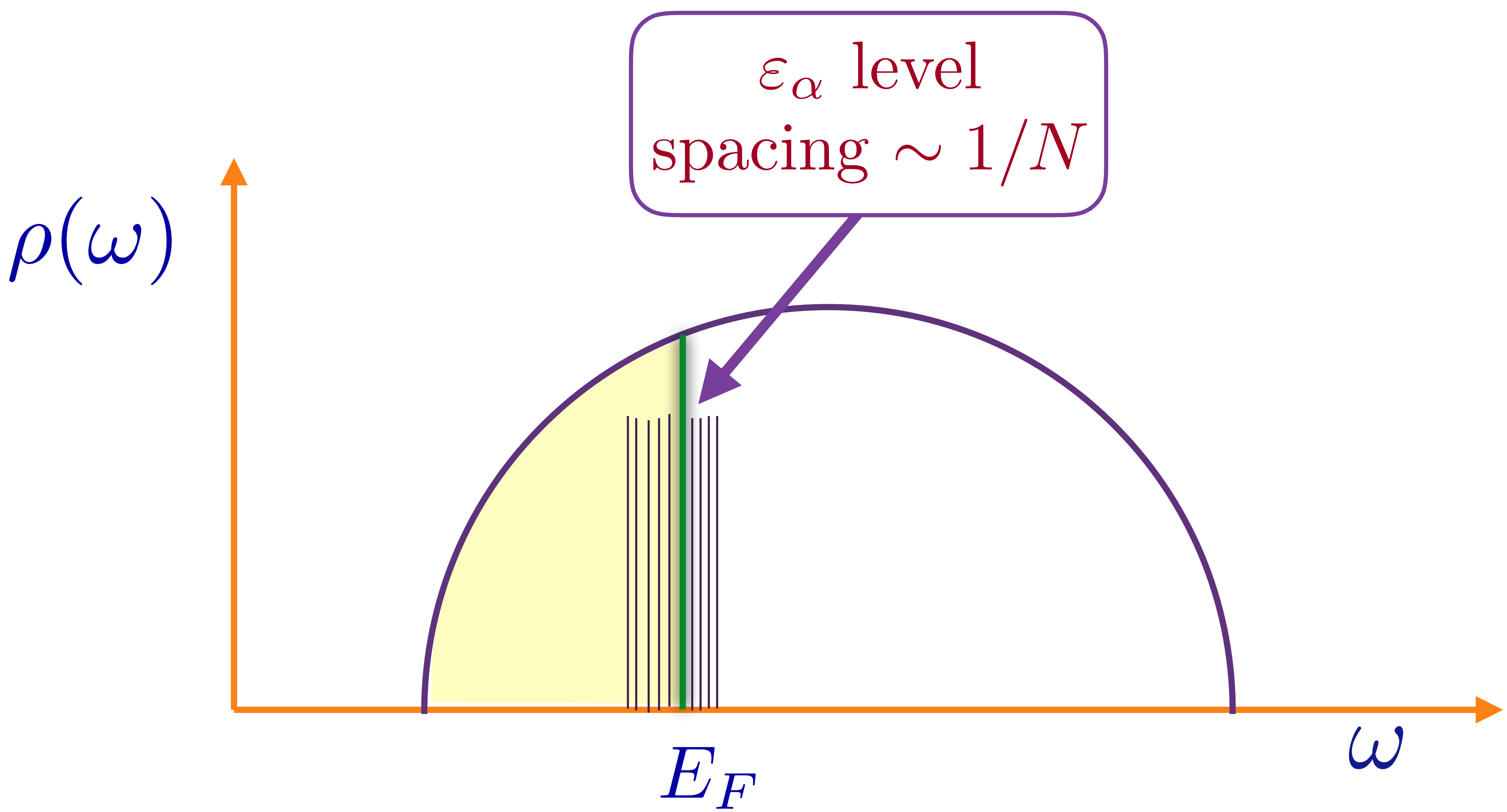

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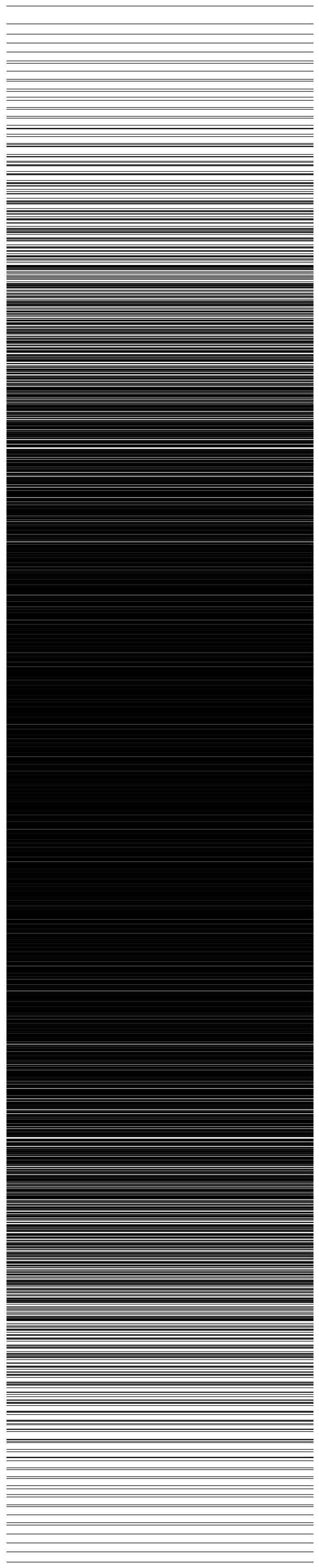

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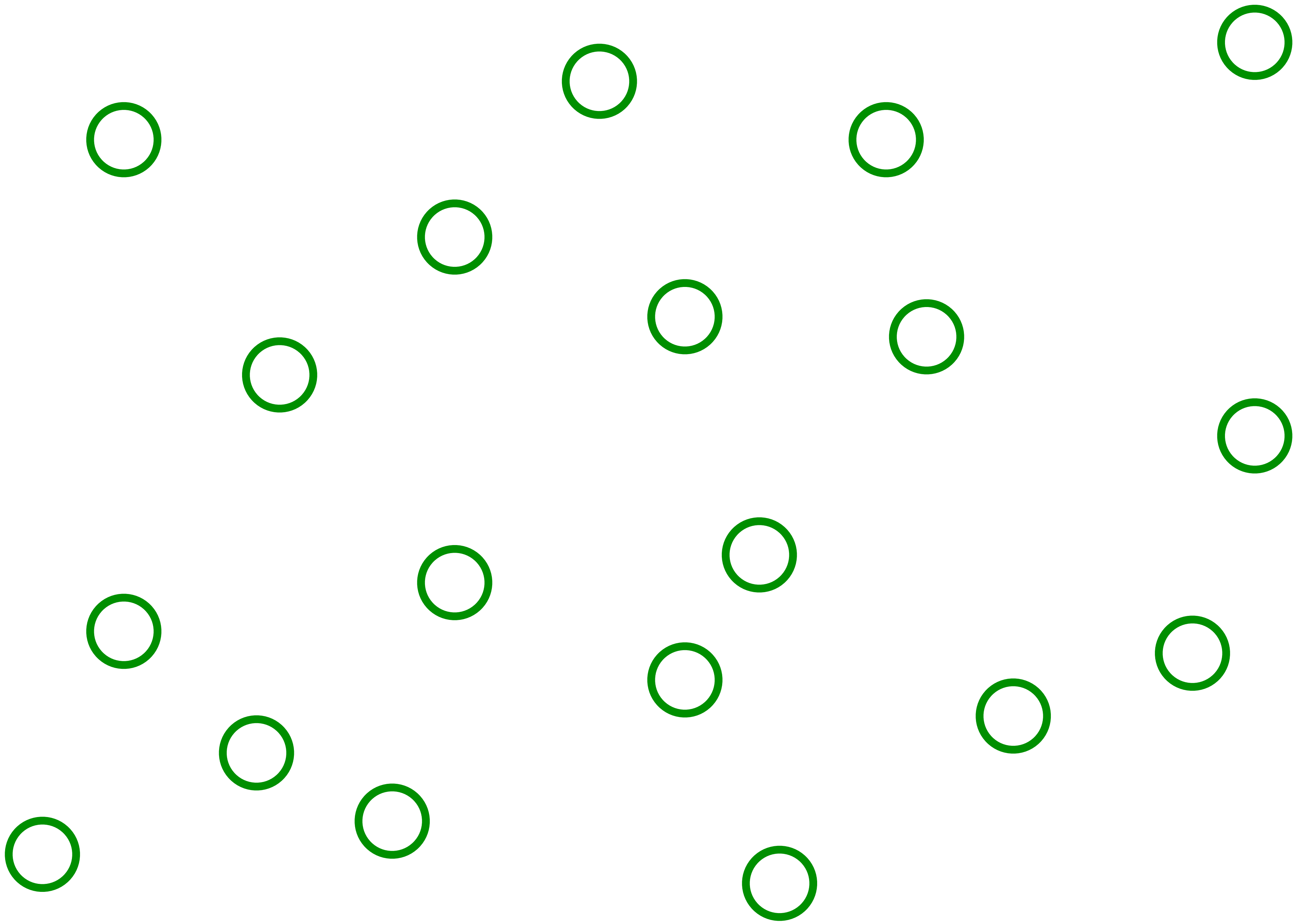
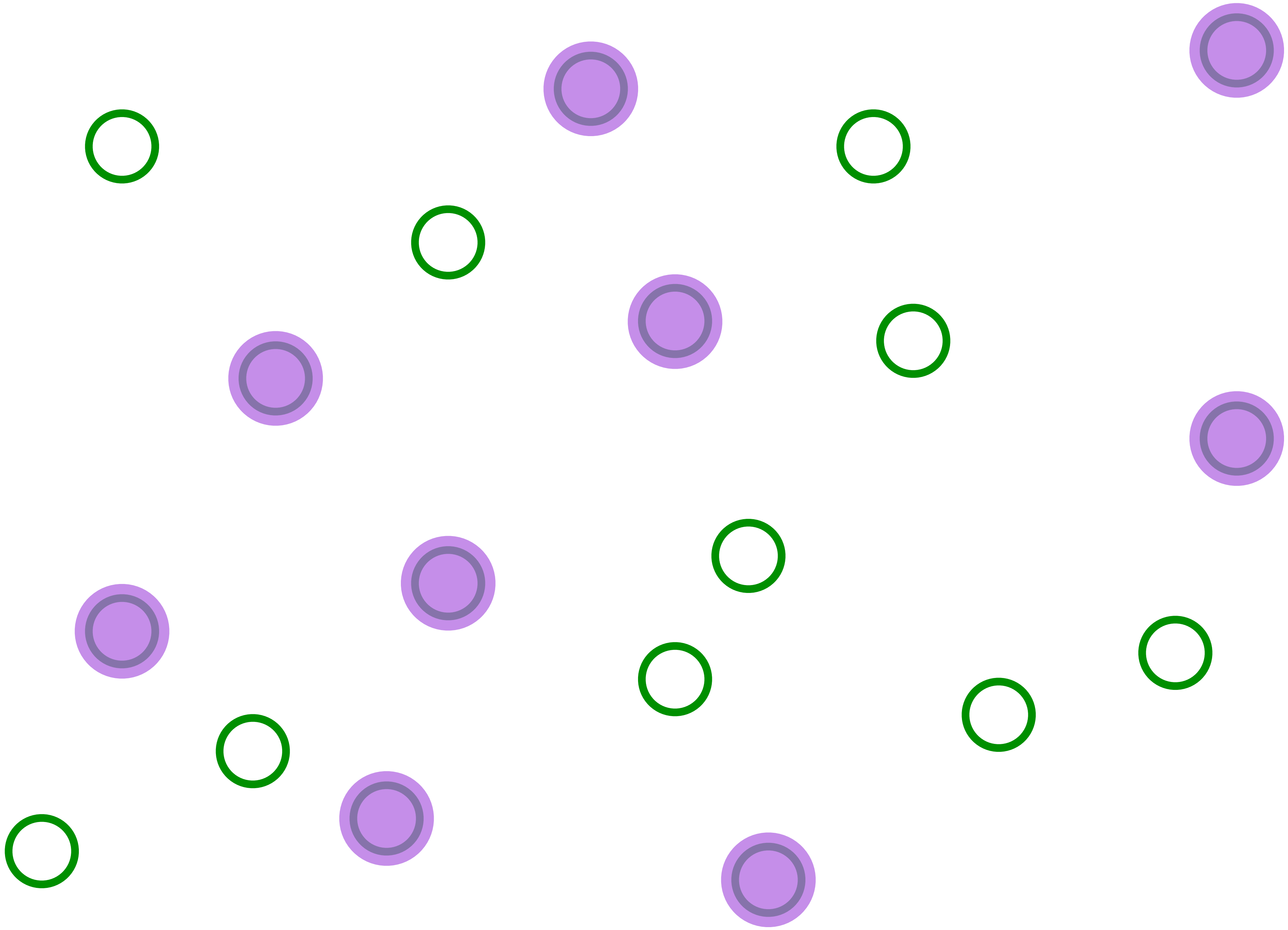
The Sachdev-Ye-Kitaev (SYK) model



Pick a set of random positions

The SYK model



Place electrons randomly on some sites

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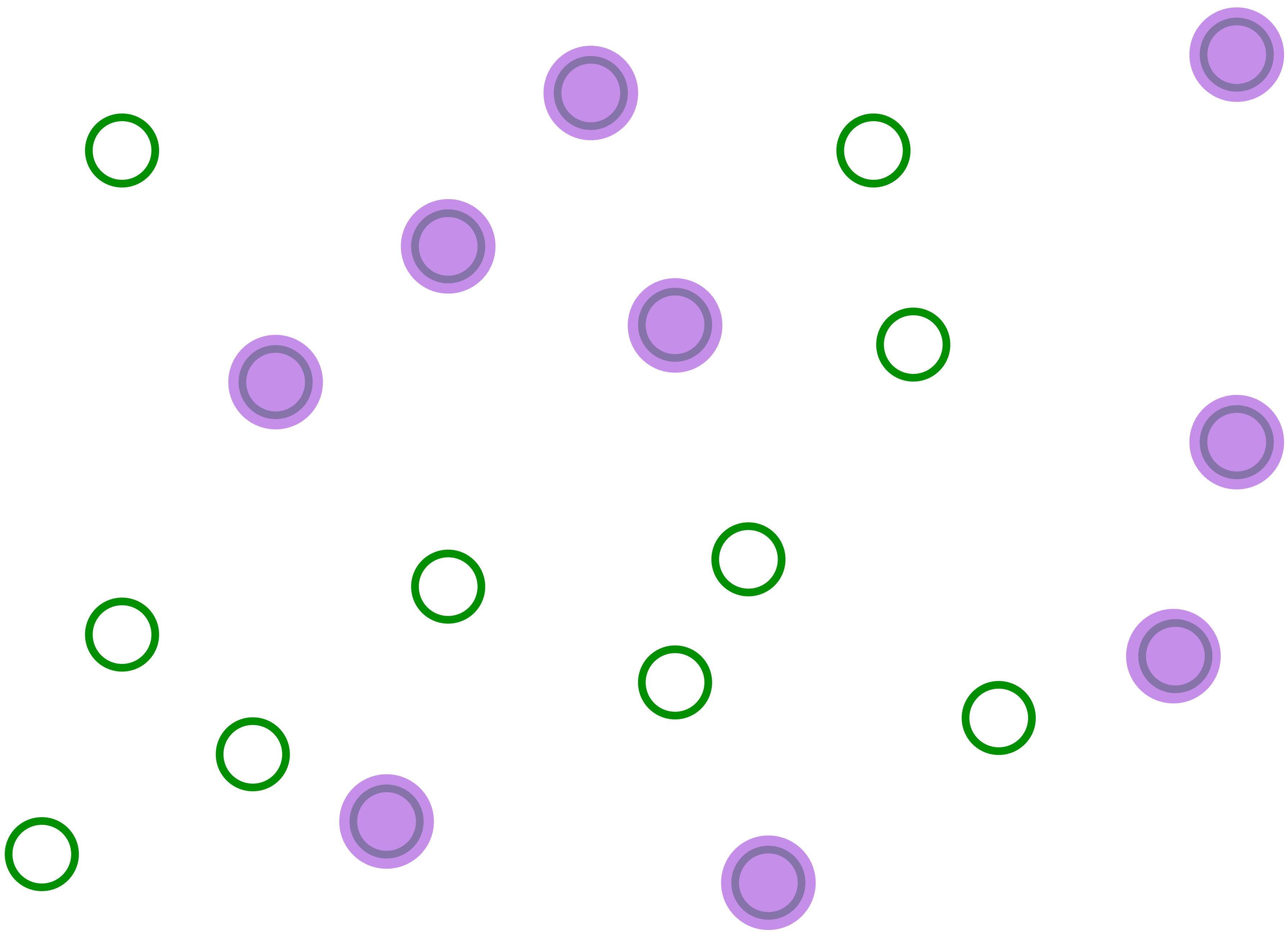
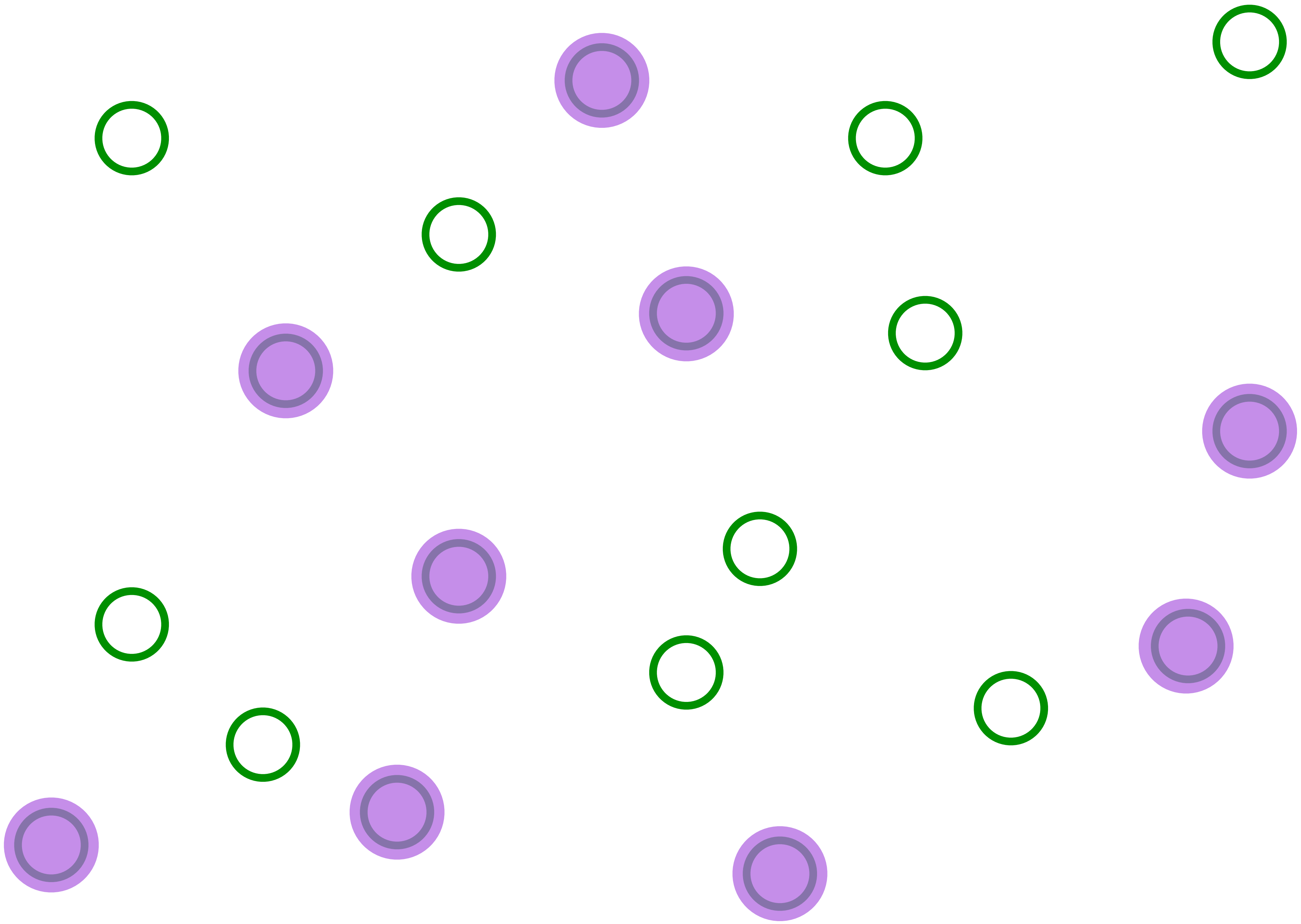


Entangle electrons pairwise randomly

The SYK model



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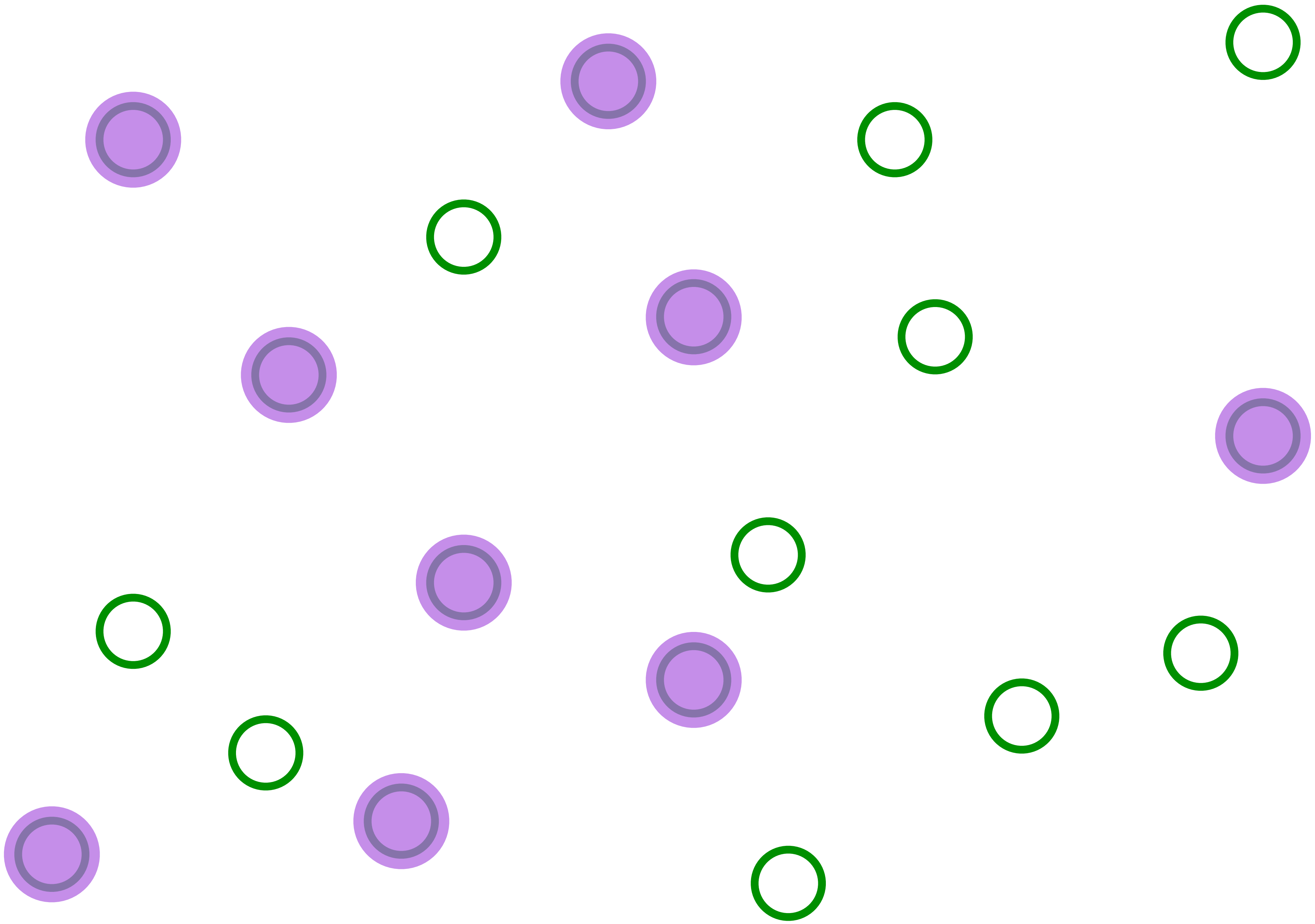

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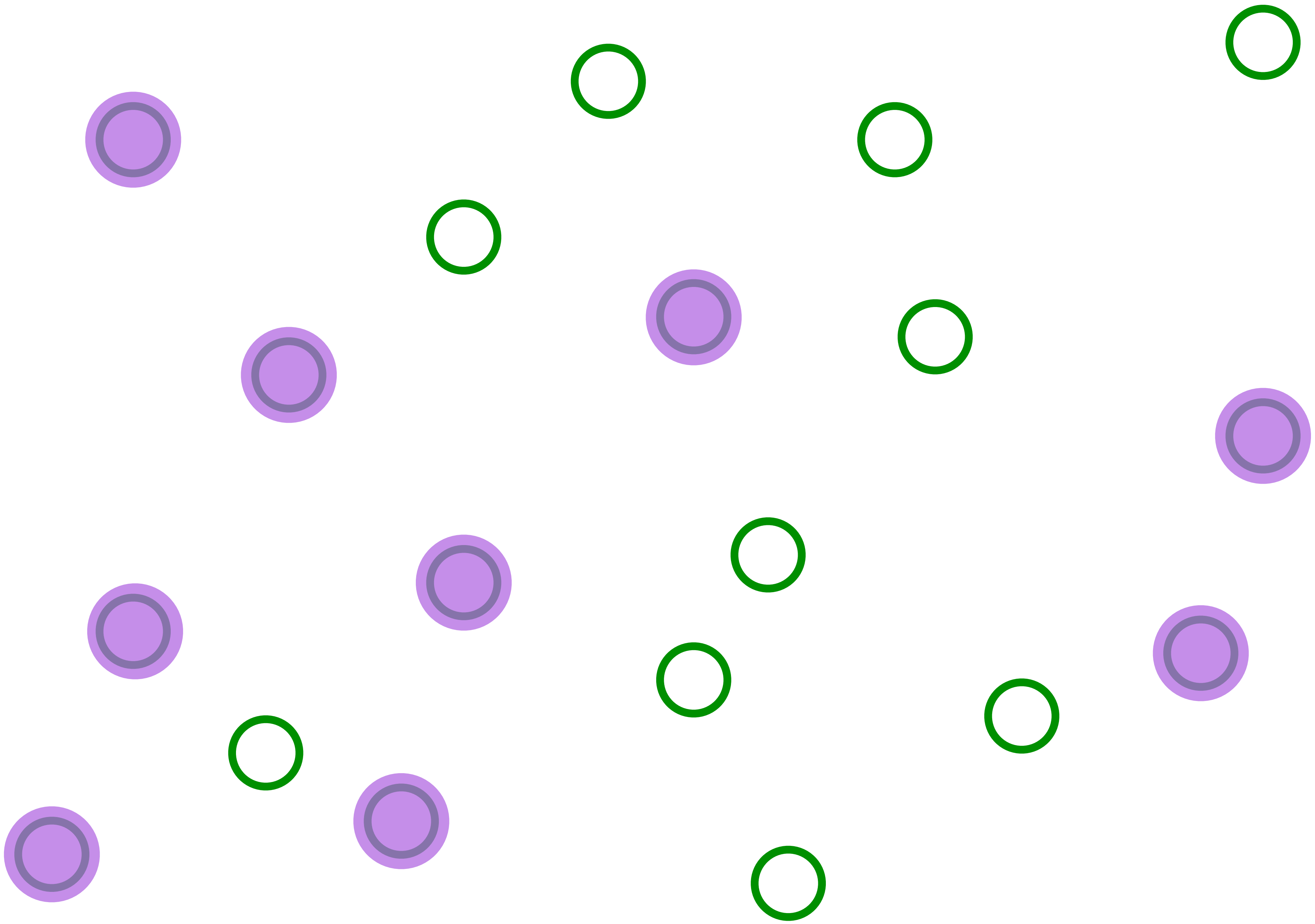

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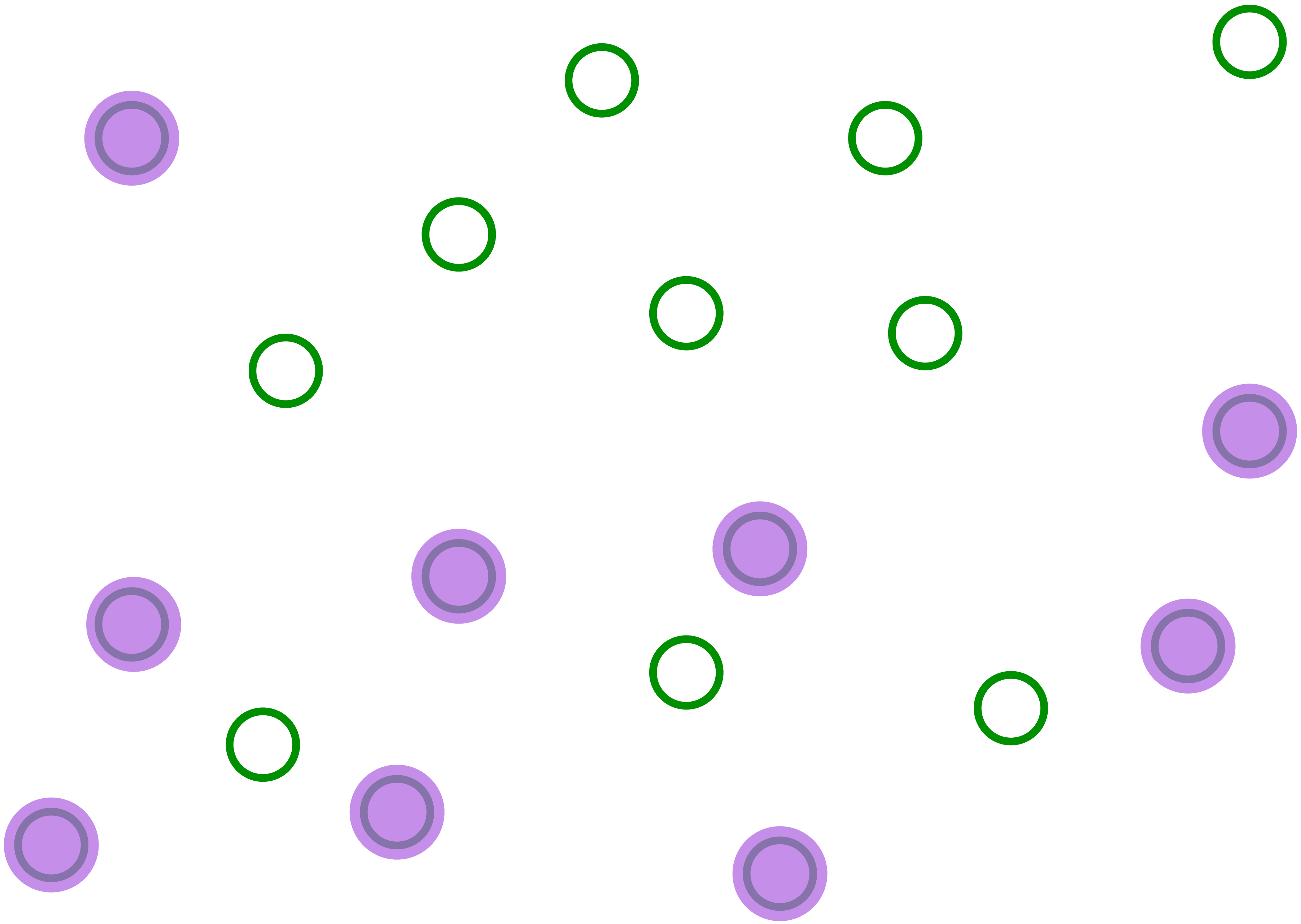

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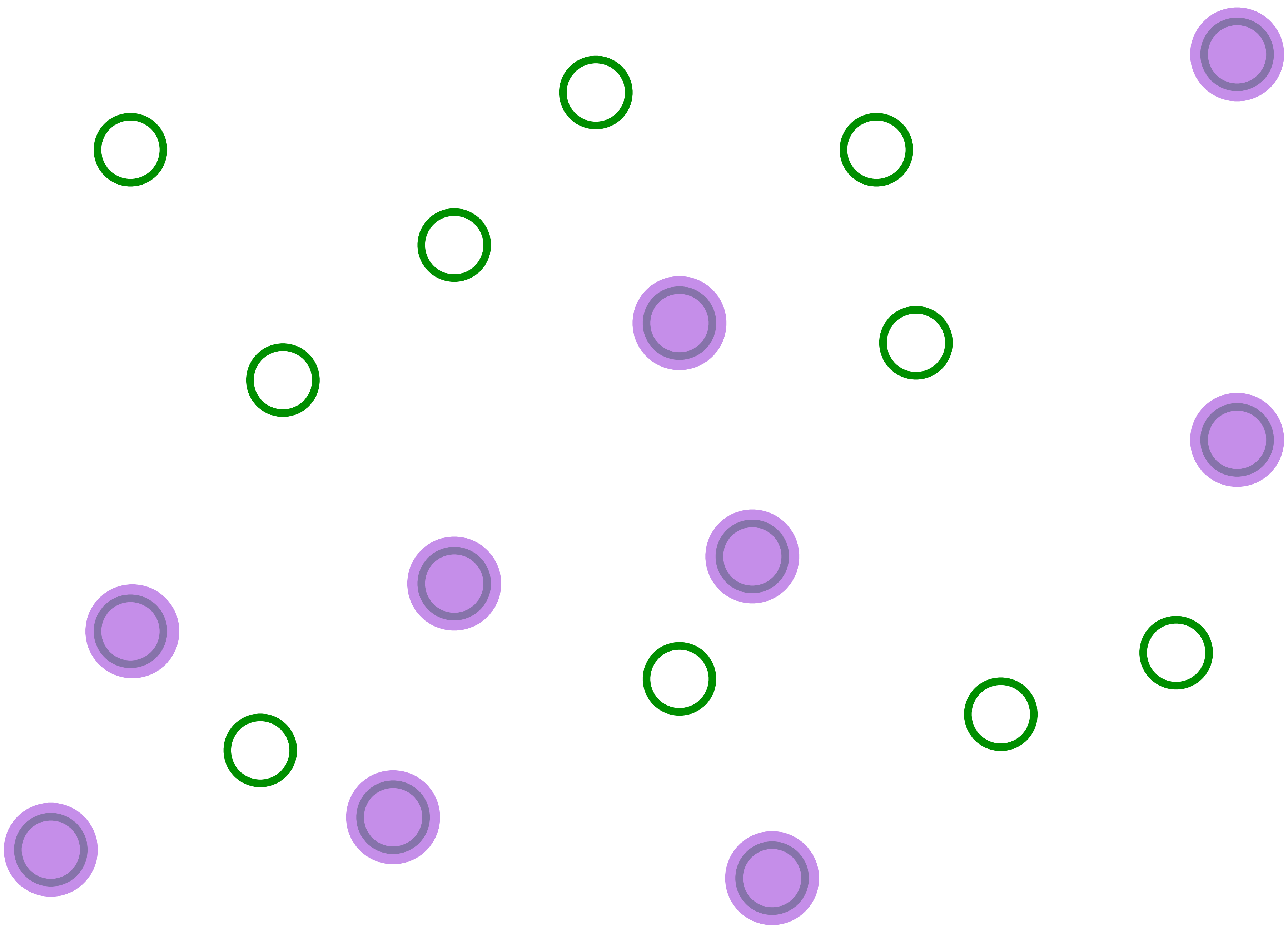

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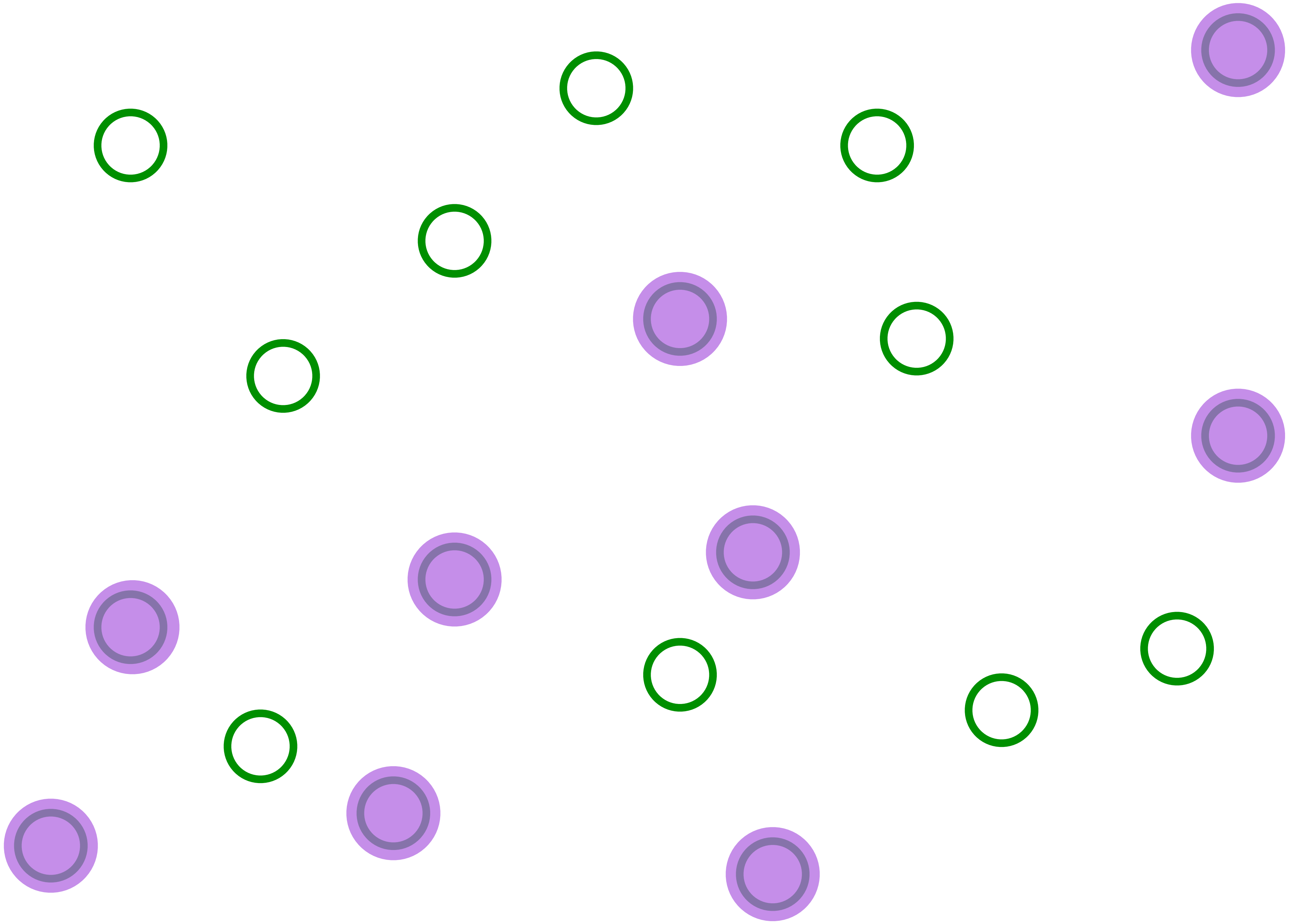


This describes both a strange metal and a black hole!

The SYK model

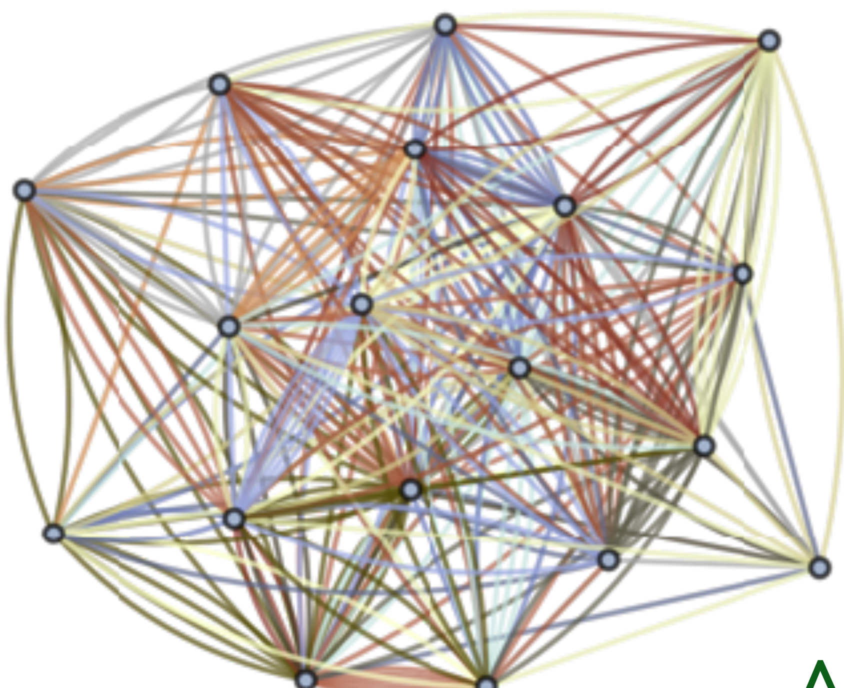
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

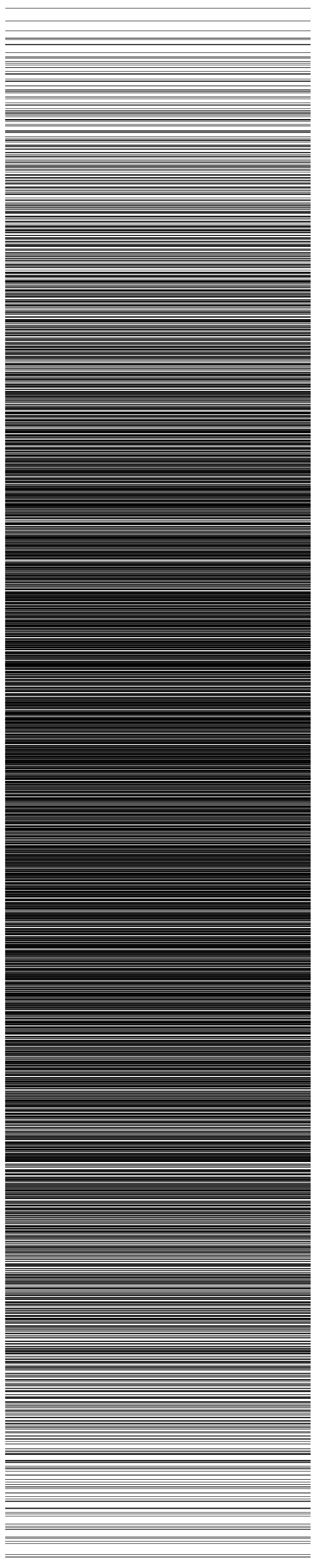
$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The SYK model



Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

There are 2^N many body levels with energy E, which do not admit a quasiparticle decomposition. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$ with

$$
\begin{aligned}
s_0 &= \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots \\
&< \ln 2
\end{aligned}
$$

where G is Catalan's constant, for the half-filled case $\mathcal{Q} = 1/2$.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

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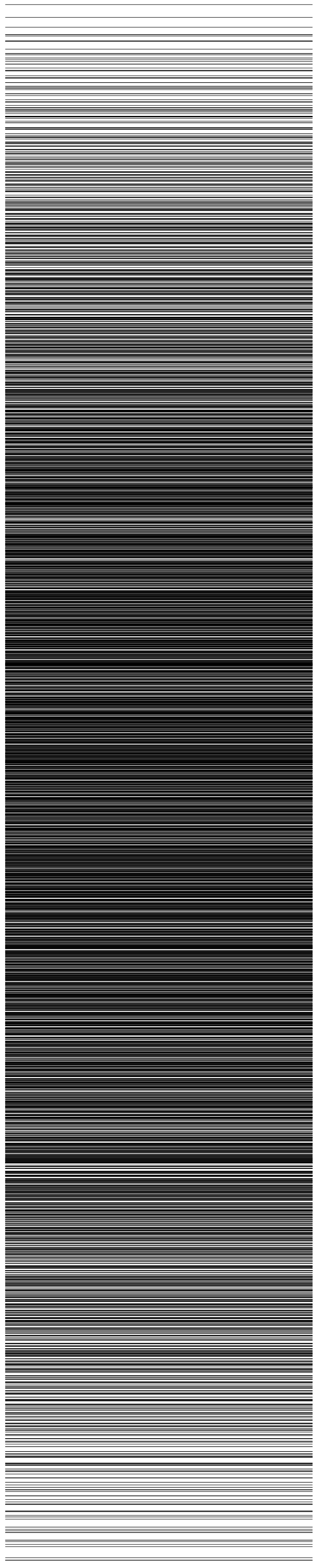

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Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

No quasiparticles !
$E \neq \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$

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Feynman graph expansion in $U_{ijk\ell}$, and graph-by-graph average, yields exact equations in the large N limit:

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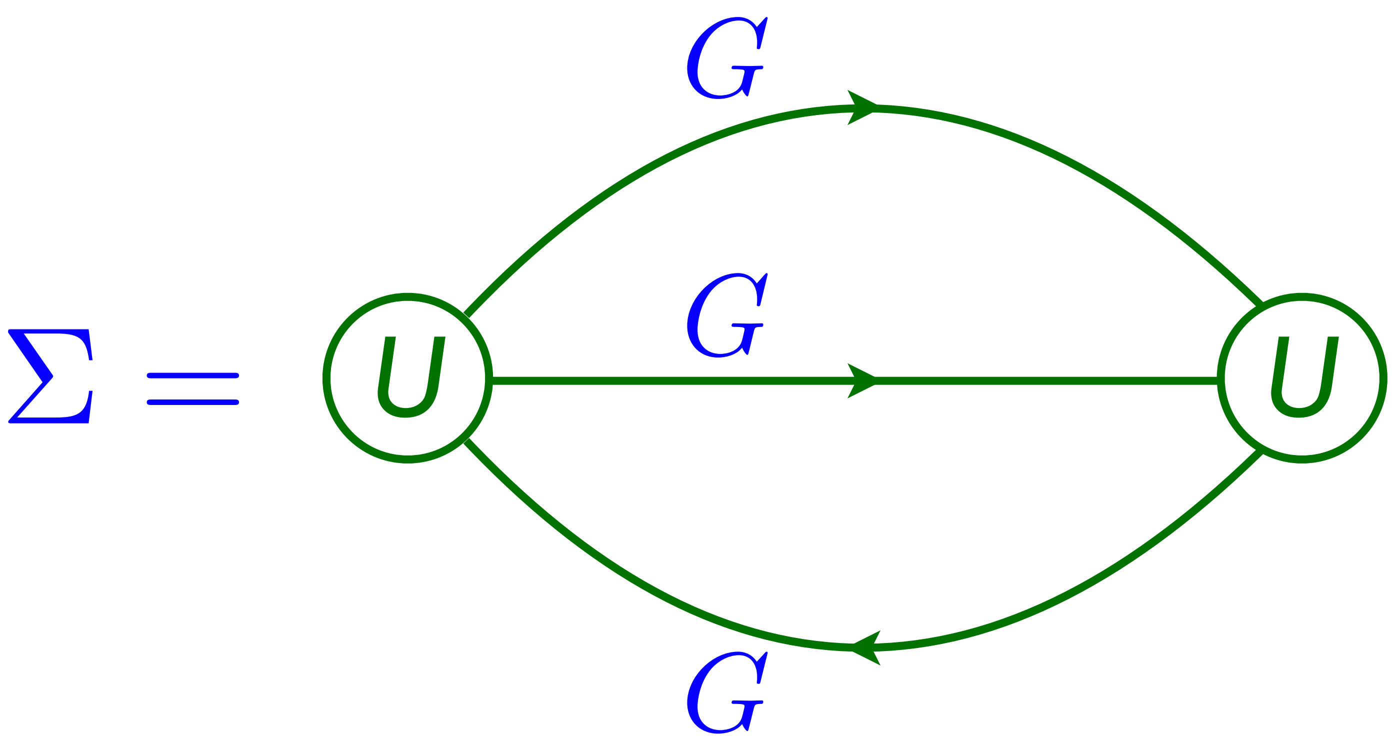

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

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$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

where $A = e^{-i\pi/4}(\pi/U^2)^{1/4}$ at half-filling. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

The SYK model

- $T = 0$ fermion Green's function is incoherent: $G(\tau) \sim \tau^{-1/2}$ at large τ. (Fermi liquids with quasiparticles have the coherent: $G(\tau) \sim 1/\tau$)

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- The last property indicates $\tau_{\text{eq}} \sim \hbar/(k_B T)$, and this has been found in a recent numerical study.

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB **96**, 205123 (2017)

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- The model exhibits eigenstate thermalization. Each eigenstate scrambles quantum information (as measured in the out-of-time-order correlation) in the fastest possible time of $\hbar/(2\pi k_B T(E))$.

J. Sonner and M. Vielma, arXiv:1707.08013

Quantum matter without quasiparticles:

- If there are no quasiparticles, then

$$E \neq \sum_{\alpha} n_{\alpha} \varepsilon_{\alpha} + \sum_{\alpha,\beta} F_{\alpha\beta} n_{\alpha} n_{\beta} + \dots$$

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- If there are no quasiparticles, then

$$\tau_{\text{eq}} = \# \frac{\hbar}{k_B T} \quad , \quad \text{the 'Planckian time'}.$$

S. Sachdev,
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Cambridge (1999)

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- If there are no quasiparticles, then

$$\tau_{\rm eq} = \# \frac{\hbar}{k_B T} \quad , \qquad \text{the `Planckian time'}.$$

- Systems without quasiparticles are the fastest possible in reaching local equilibrium, and all many-body quantum systems obey, as $T \to 0$

$$\tau_{\rm eq} > C \frac{\hbar}{k_B T} \,.$$

S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

  - In Fermi liquids $\tau_{\rm eq} \sim 1/T^2$, and so the bound is obeyed as $T \to 0$.
  - This bound rules out quantum systems with *e.g.* $\tau_{\rm eq} \sim \hbar/(J k_B T)^{1/2}$.
  - There is no bound in classical mechanics ($\hbar \to 0$). By cranking up frequencies, we can attain equilibrium as quickly as we desire.

SYK models and black holes

- Black holes have an entropy proportional to their surface area, and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.
- Black holes relax to thermal equilibrium in a time $\sim \hbar/(k_B T_H) = 8\pi G M/c^3$.

Black holes

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- Black holes in $d+1$ spatial dimensions are similar to a quantum system without quasiparticles in d spatial dimensions.

Black holes

SYK models and black holes

PHYSICAL REVIEW LETTERS **105**, 151602 (2010)

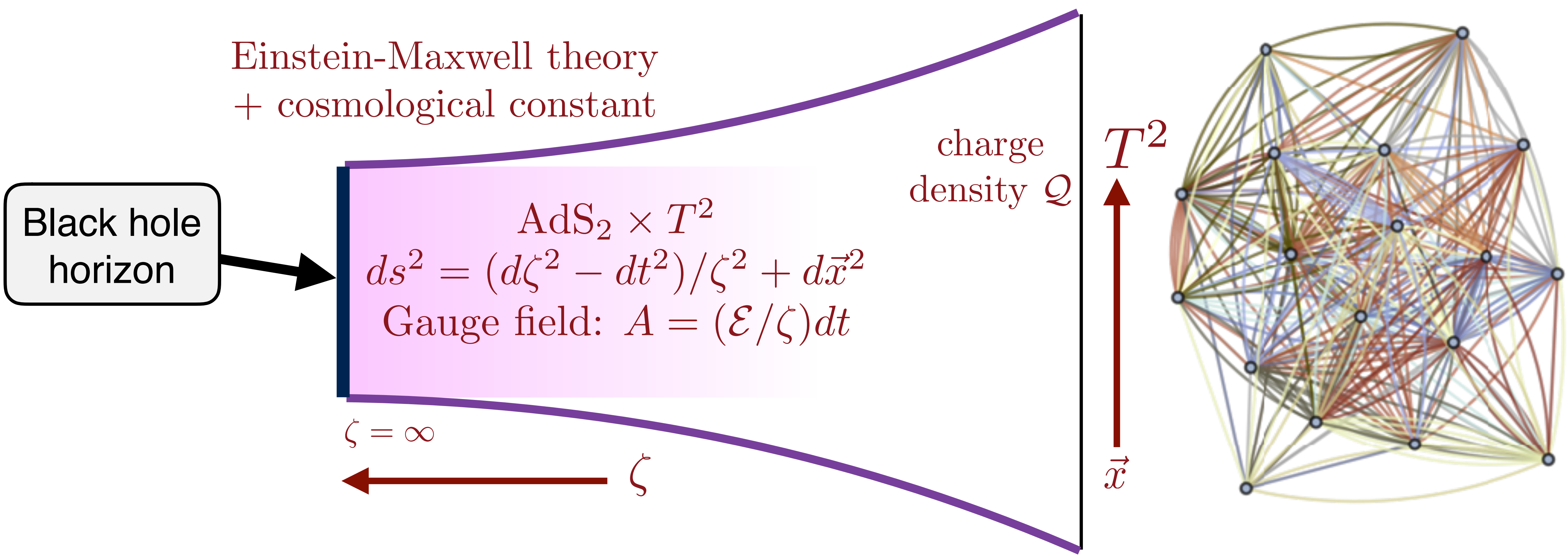
Holographic Metals and the Fractionalized Fermi Liquid

Subir Sachdev

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 23 June 2010; published 4 October 2010)

We show that there is a close correspondence between the physical properties of holographic metals near charged black holes in anti–de Sitter (AdS) space, and the fractionalized Fermi liquid phase of the lattice Anderson model. The latter phase has a "small" Fermi surface of conduction electrons, along with a spin liquid of local moments. This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, $AdS_2 \times R^2$ physics of Reissner-Nordström black holes.



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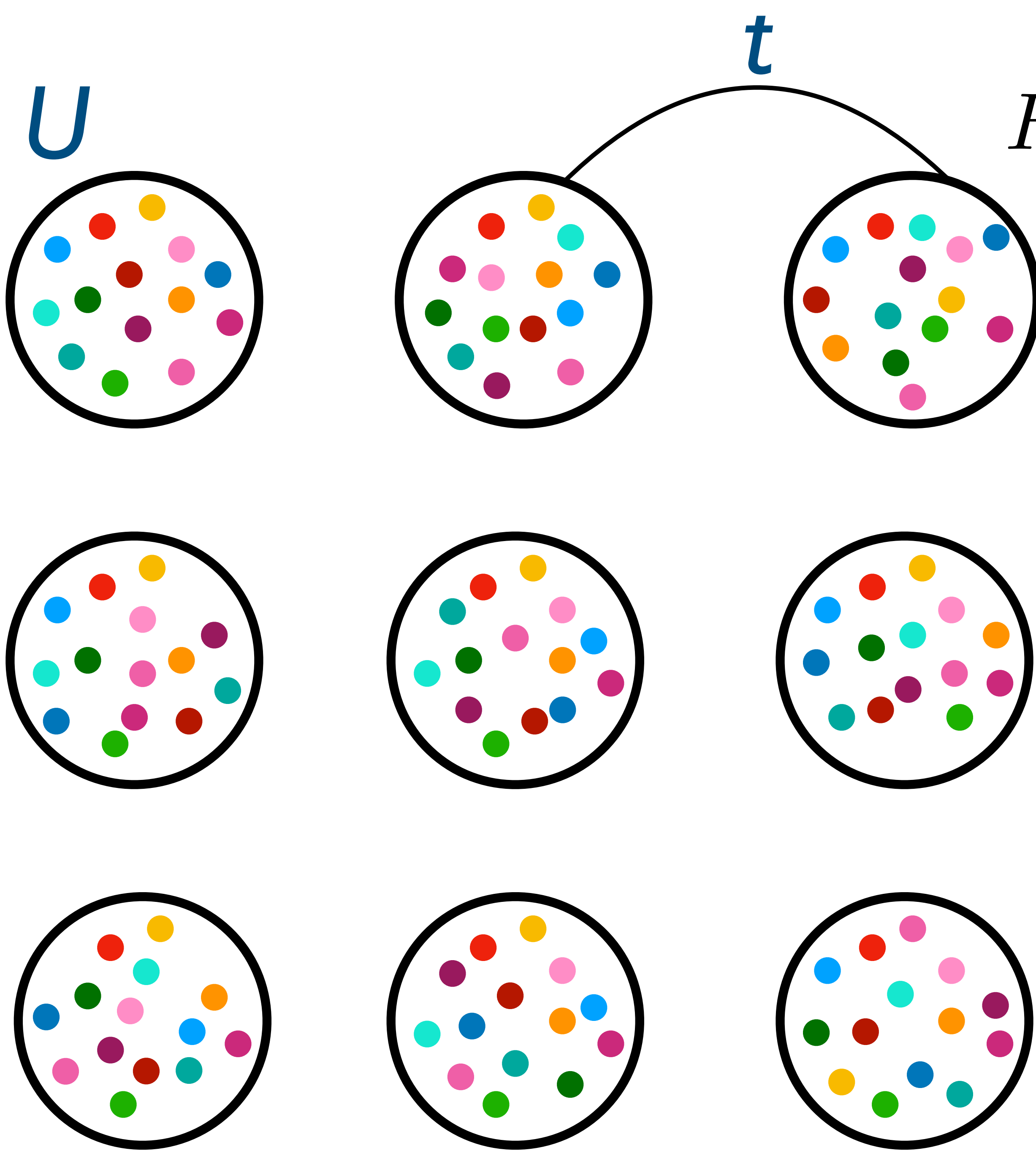
SYK building blocks for a strange metal

SYK quantum islands of electrons with random hopping between them.



$$H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$

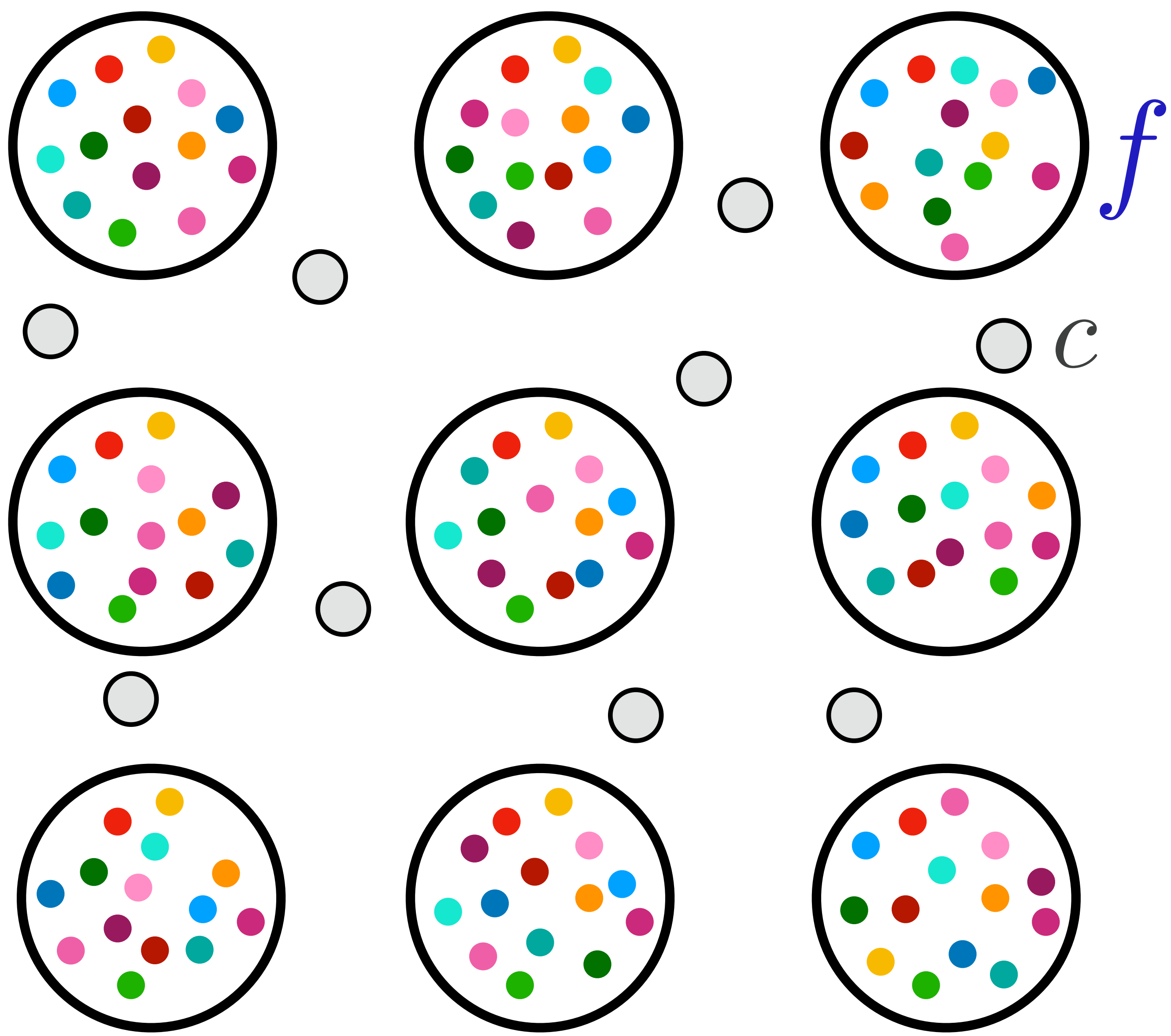
$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

SYK building blocks for a strange metal

Mobile electrons (c) interacting with SYK quantum islands (f) with random exchange interactions.

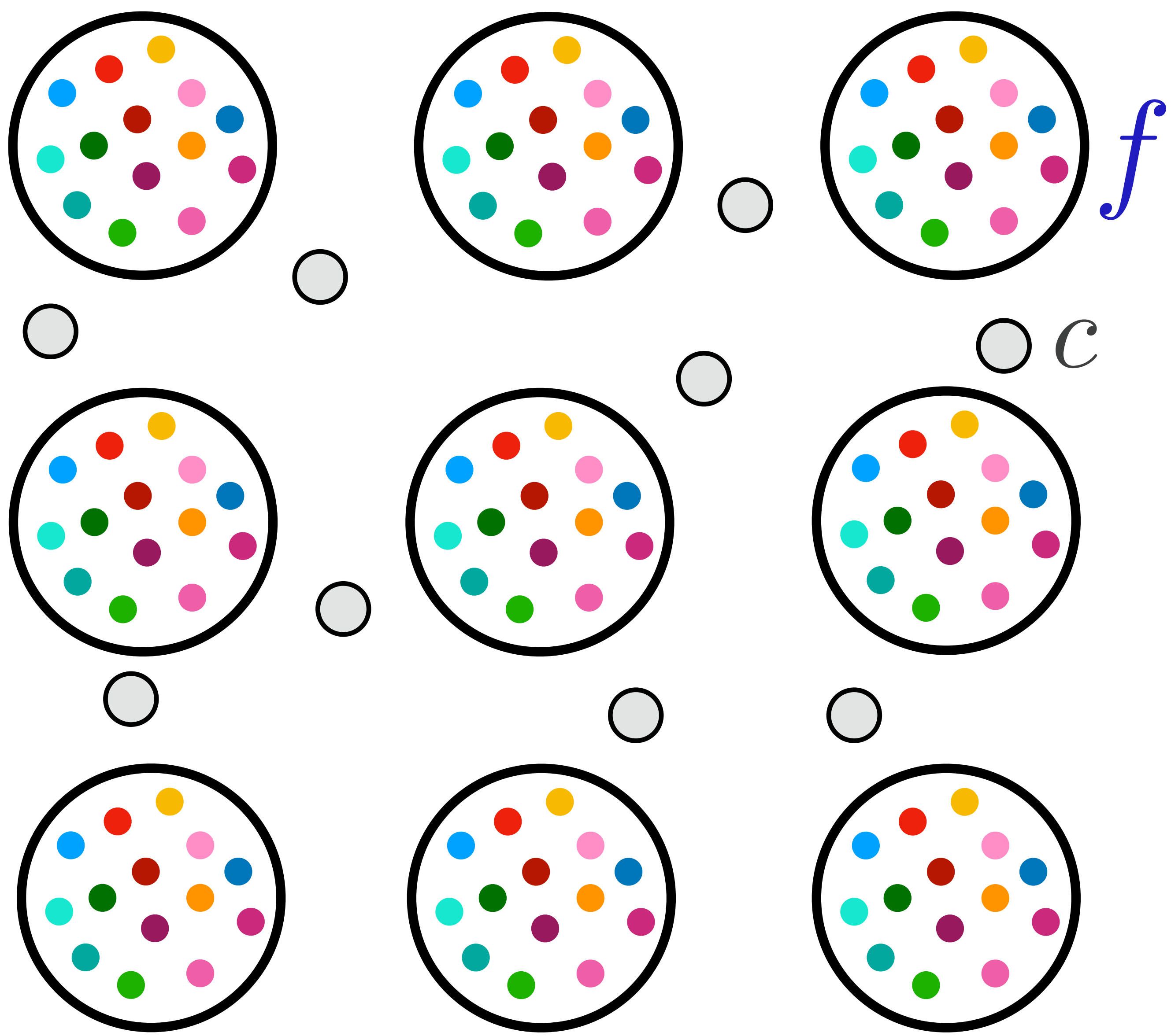
This yields the first model agreeing with magnetotransport in strange metals



Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

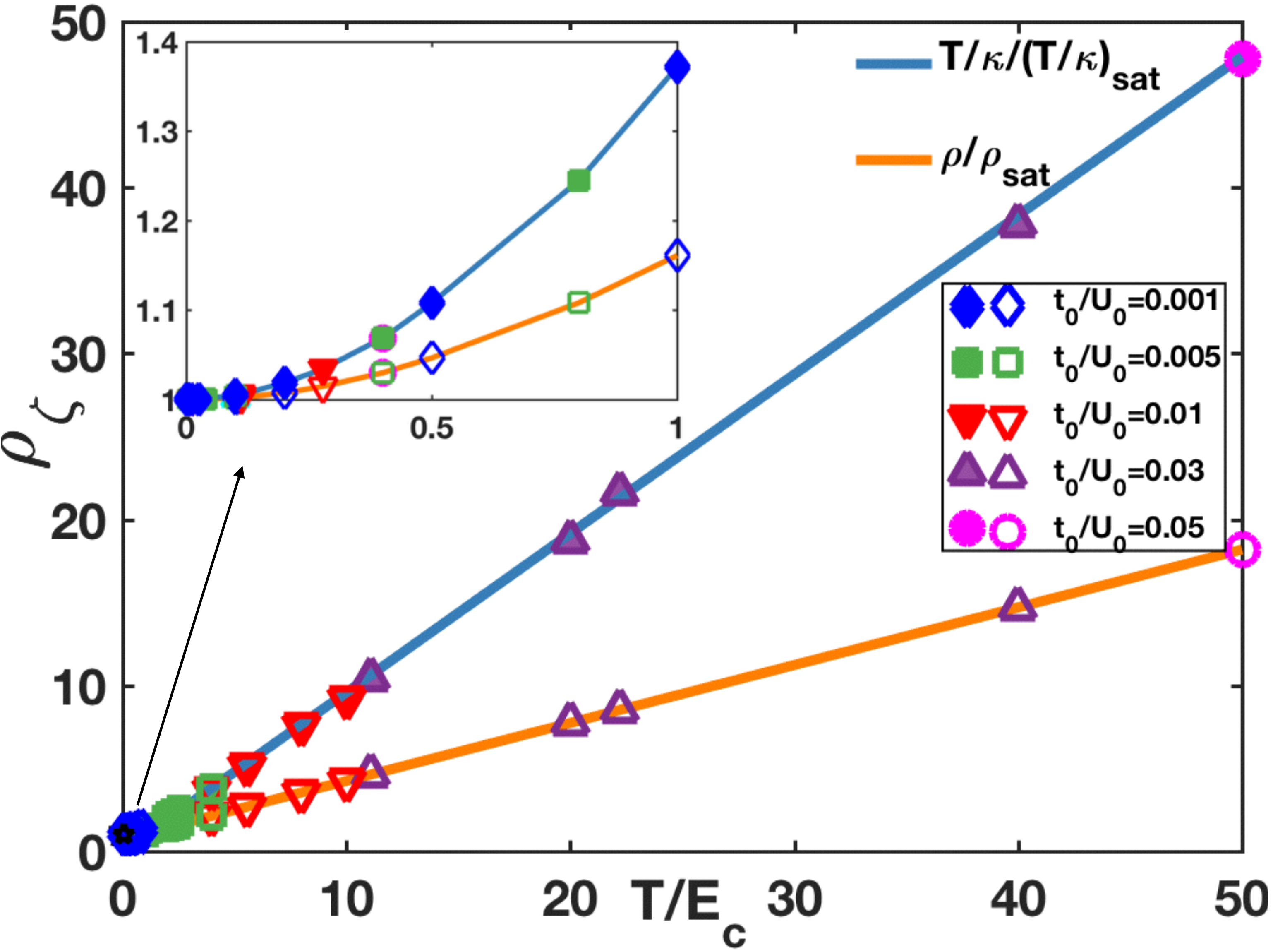
SYK building blocks for a strange metal

Mobile electrons (*c*) interacting with SYK quantum islands (*f*) with non-random exchange interactions.



Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, arXiv:1801.06178

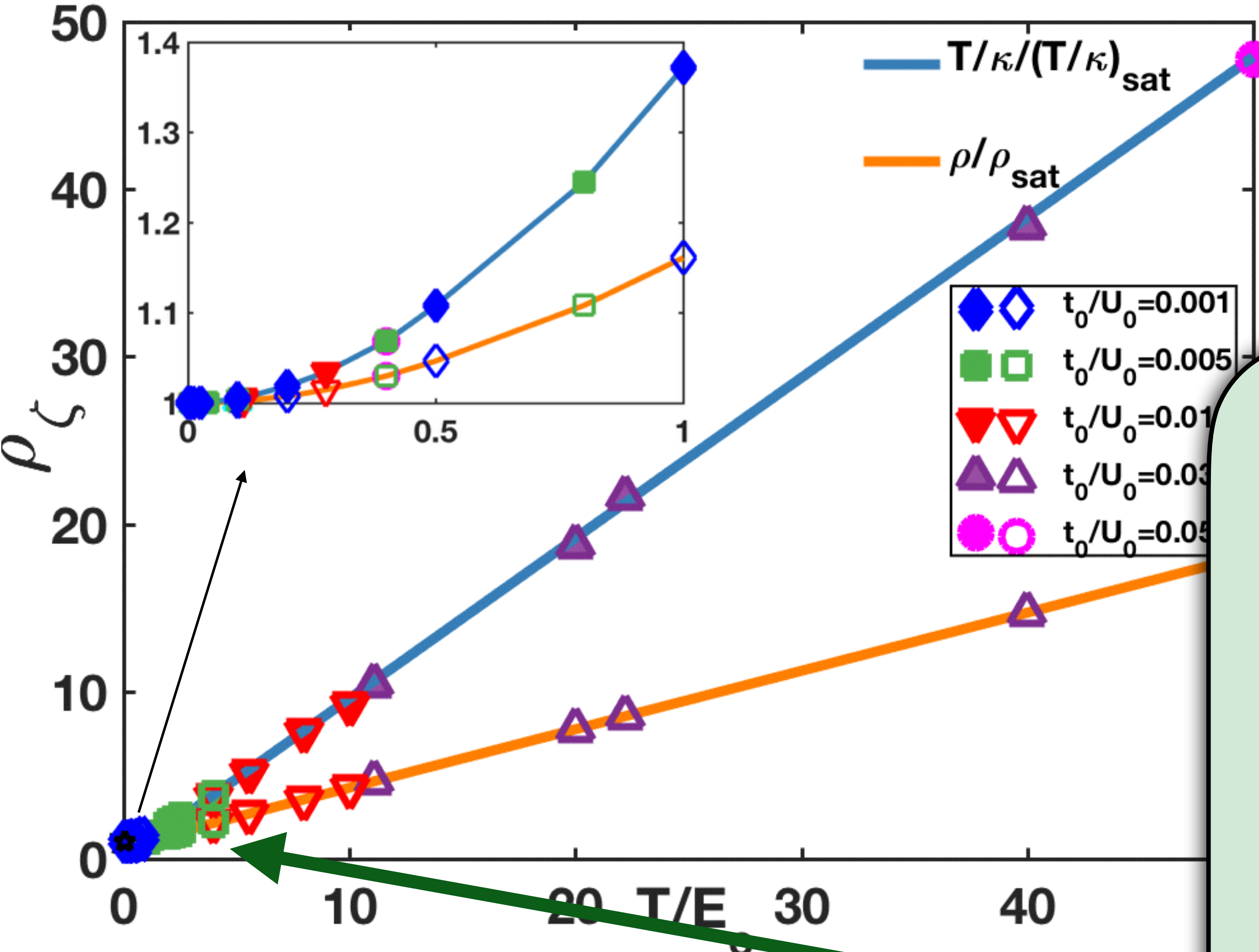
Low 'coherence' scale



$$E_c \sim \frac{t_0^2}{U}$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Low ‘coherence’ scale



$$E_c \sim \frac{t_0^2}{U}$$

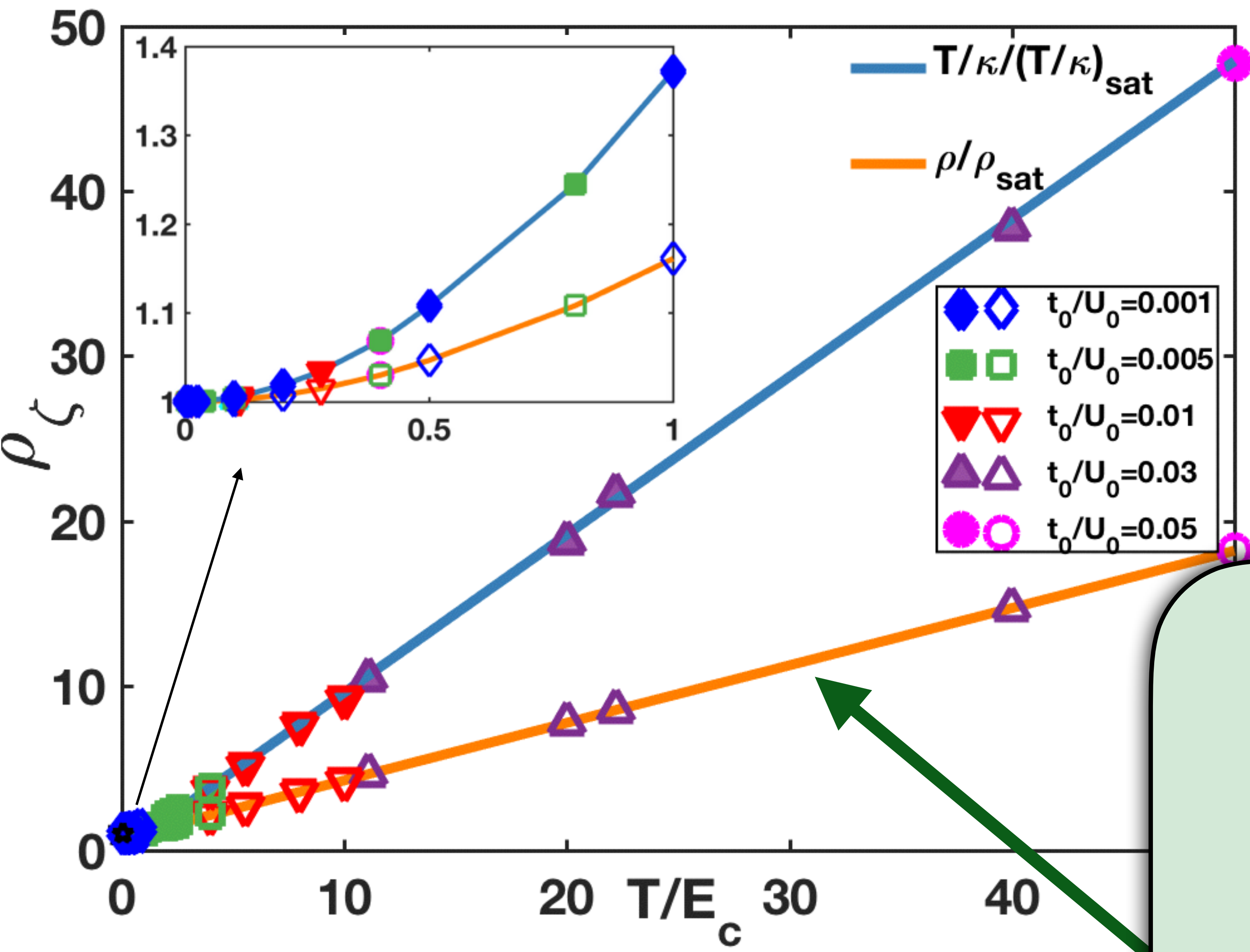
For $T < E_c$, the resistivity, ρ, and entropy density, s, are

$$\rho = \frac{h}{e^2}\left[c_1 + c_2\left(\frac{T}{E_c}\right)^2\right]$$

$$s \sim s_0\left(\frac{T}{E_c}\right)$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Low ‘coherence’ scale

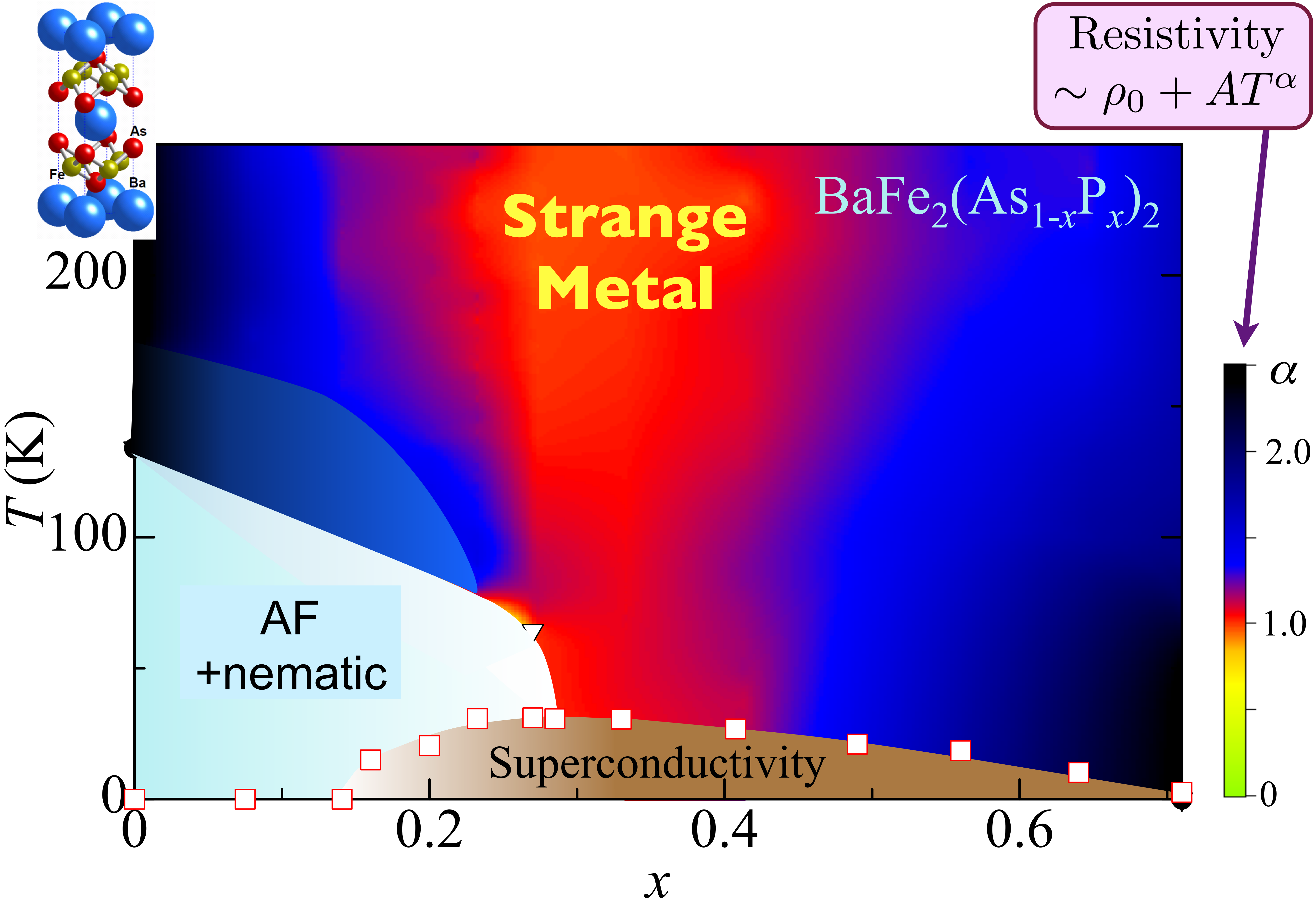


$$E_c \sim \frac{t_0^2}{U}$$

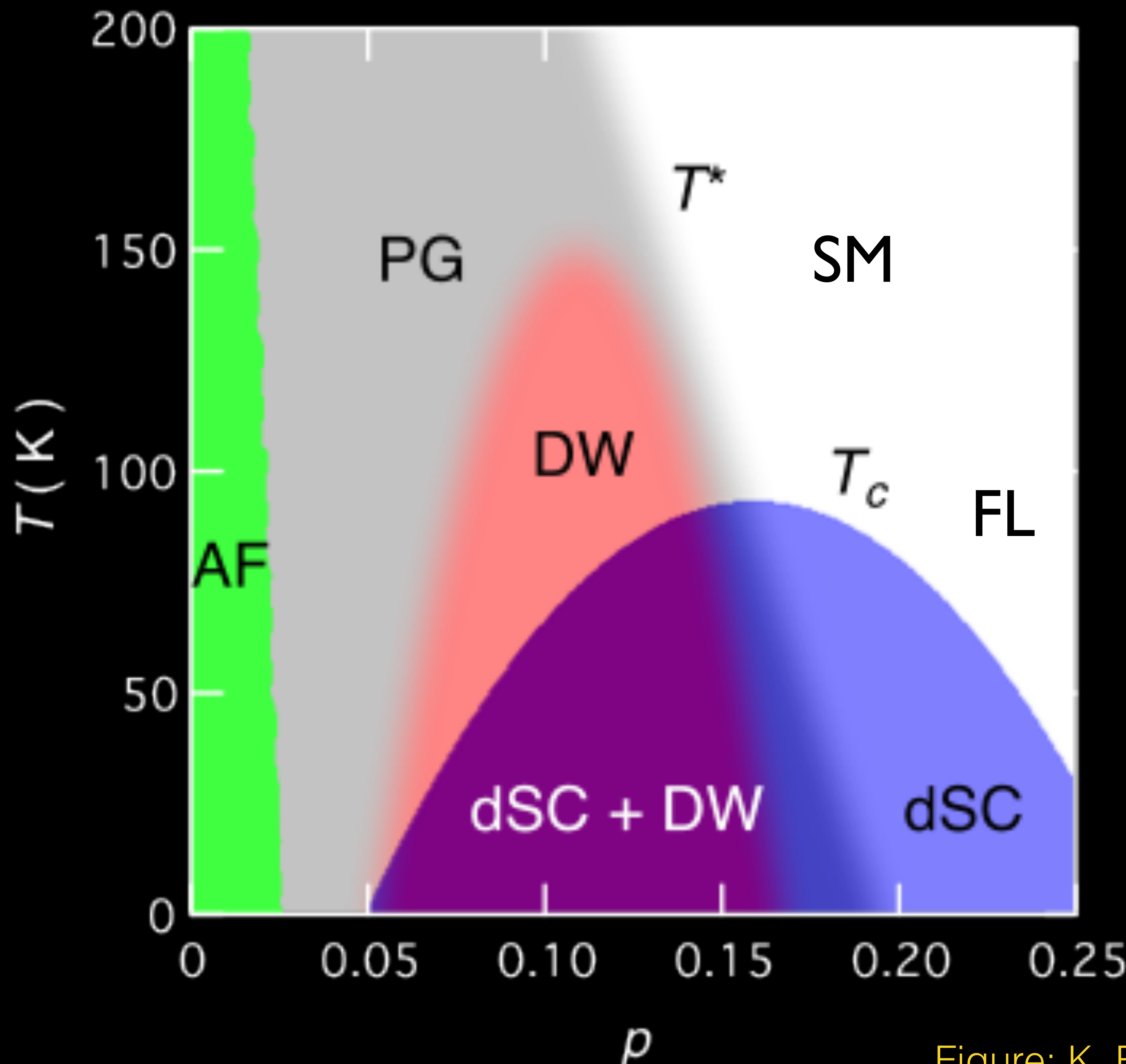
For $E_c < T < U$, the resistivity, ρ, and entropy density, s, are

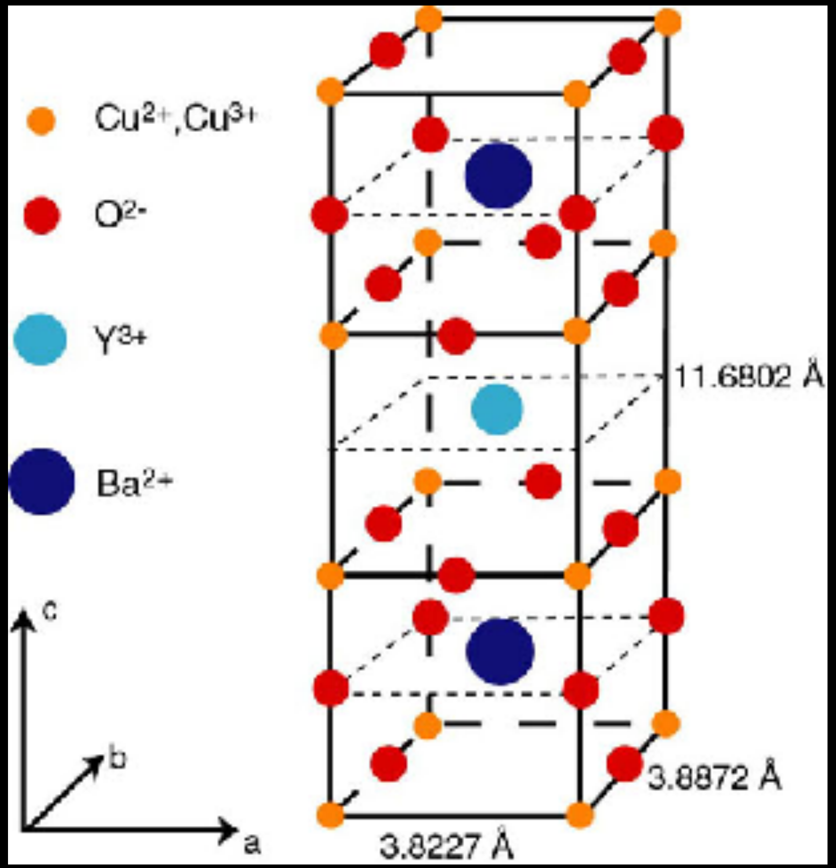
$$\rho \sim \frac{h}{e^2}\left(\frac{T}{E_c}\right) \, , \quad s = s_0$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)





$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

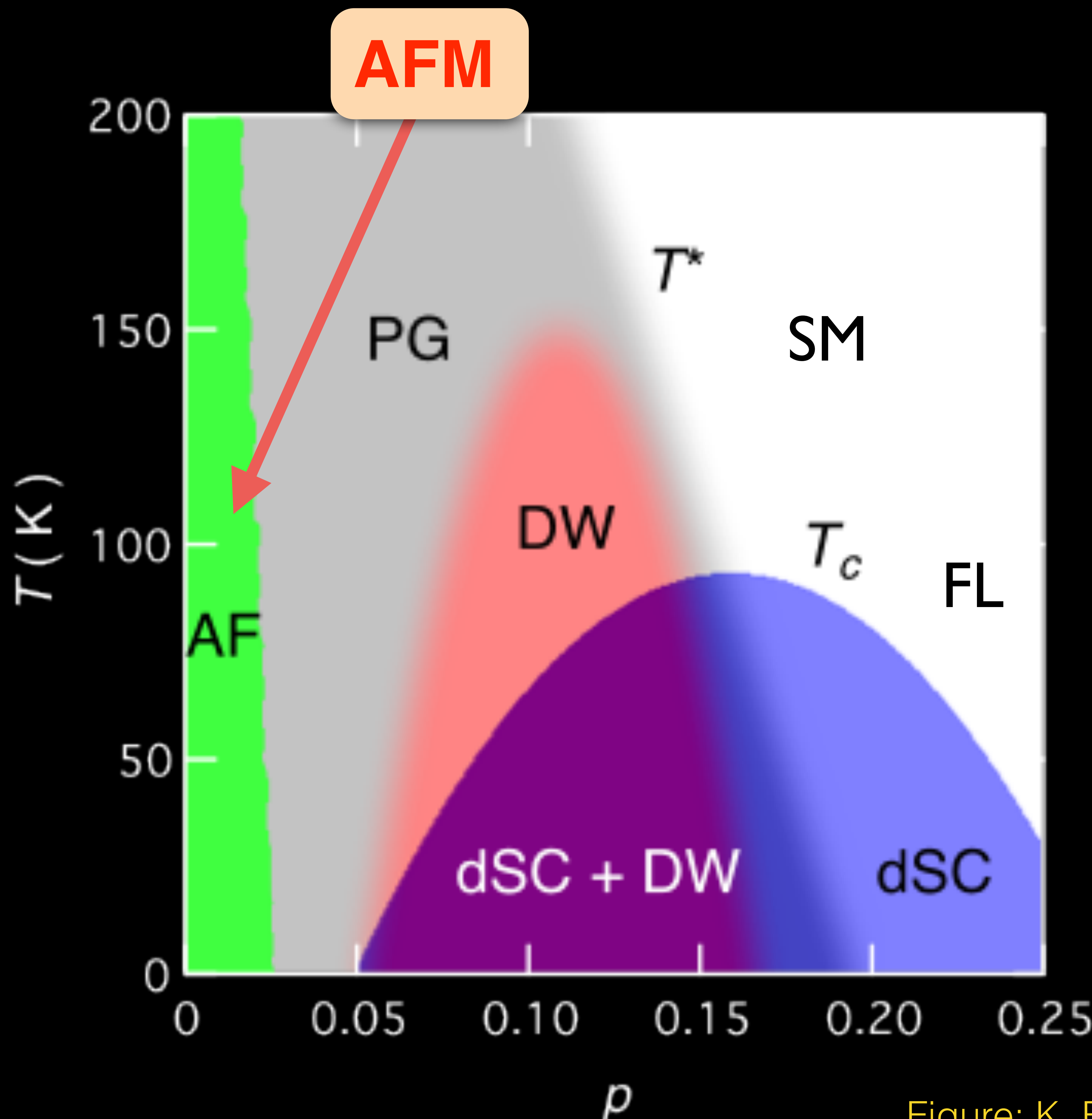
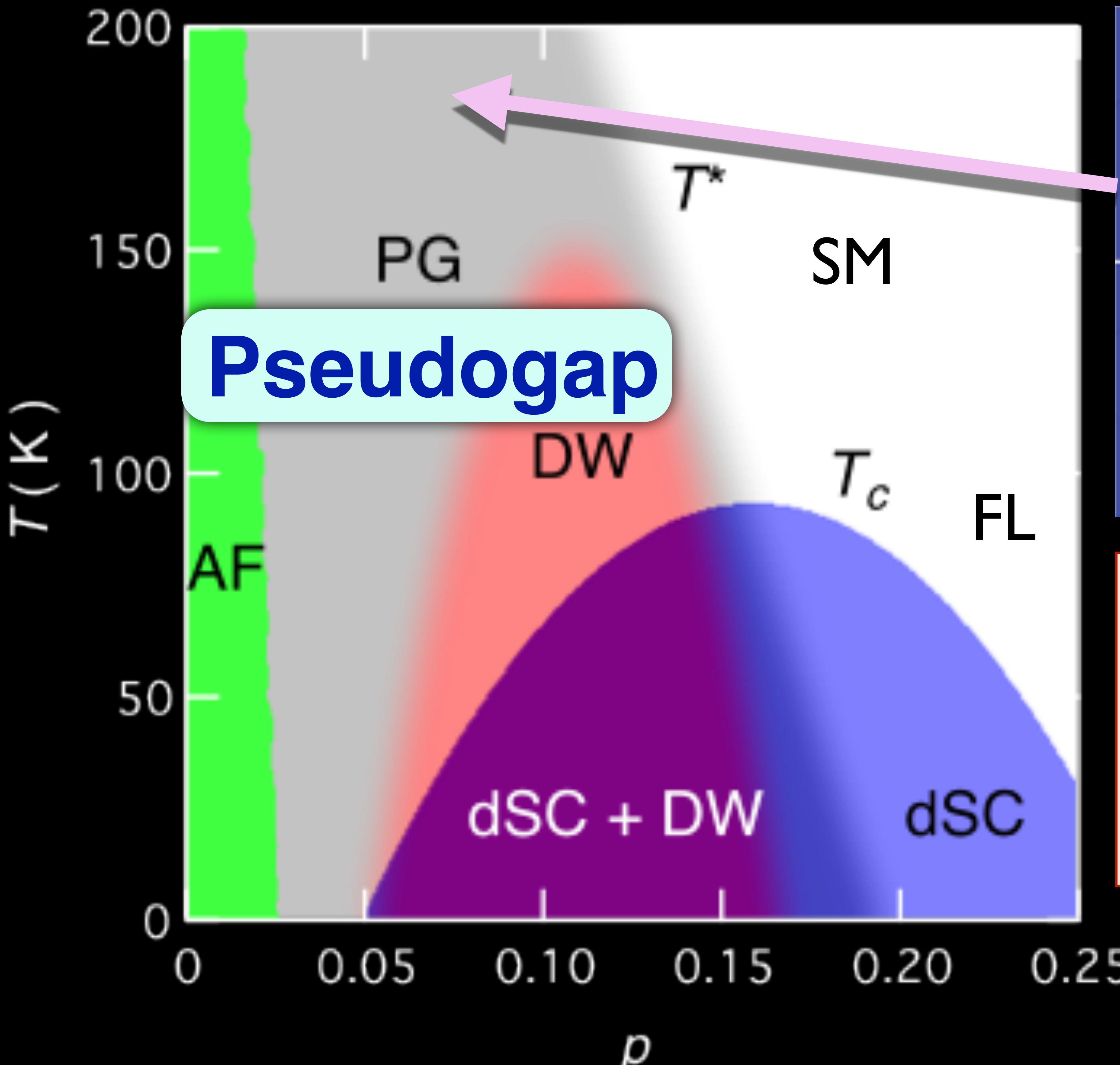
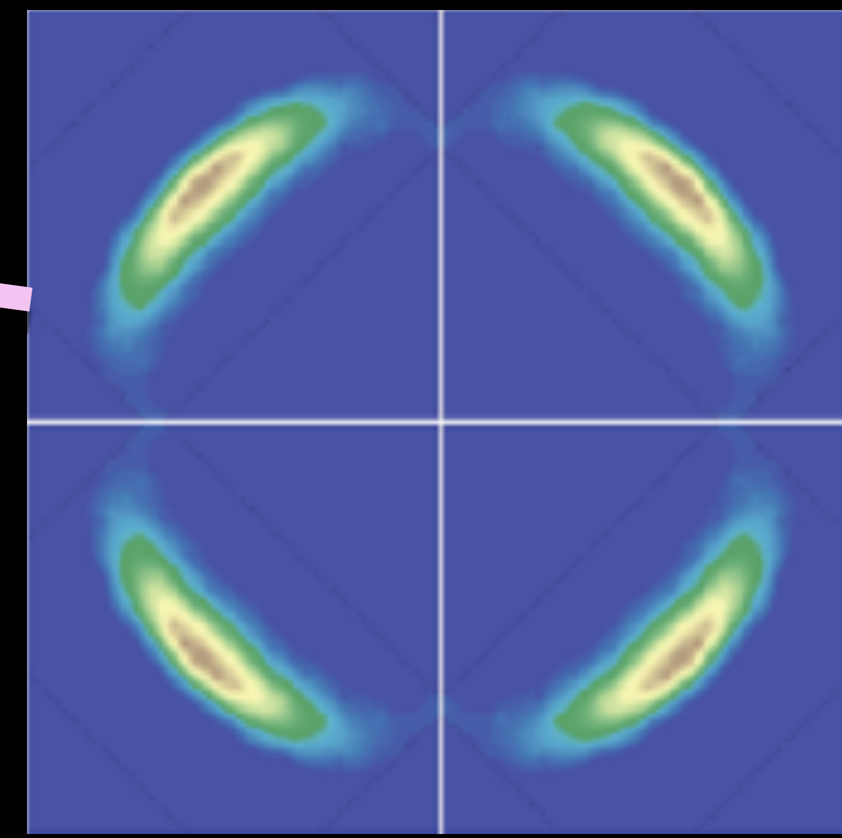


Figure: K. Fujita and J. C. Seamus Davis

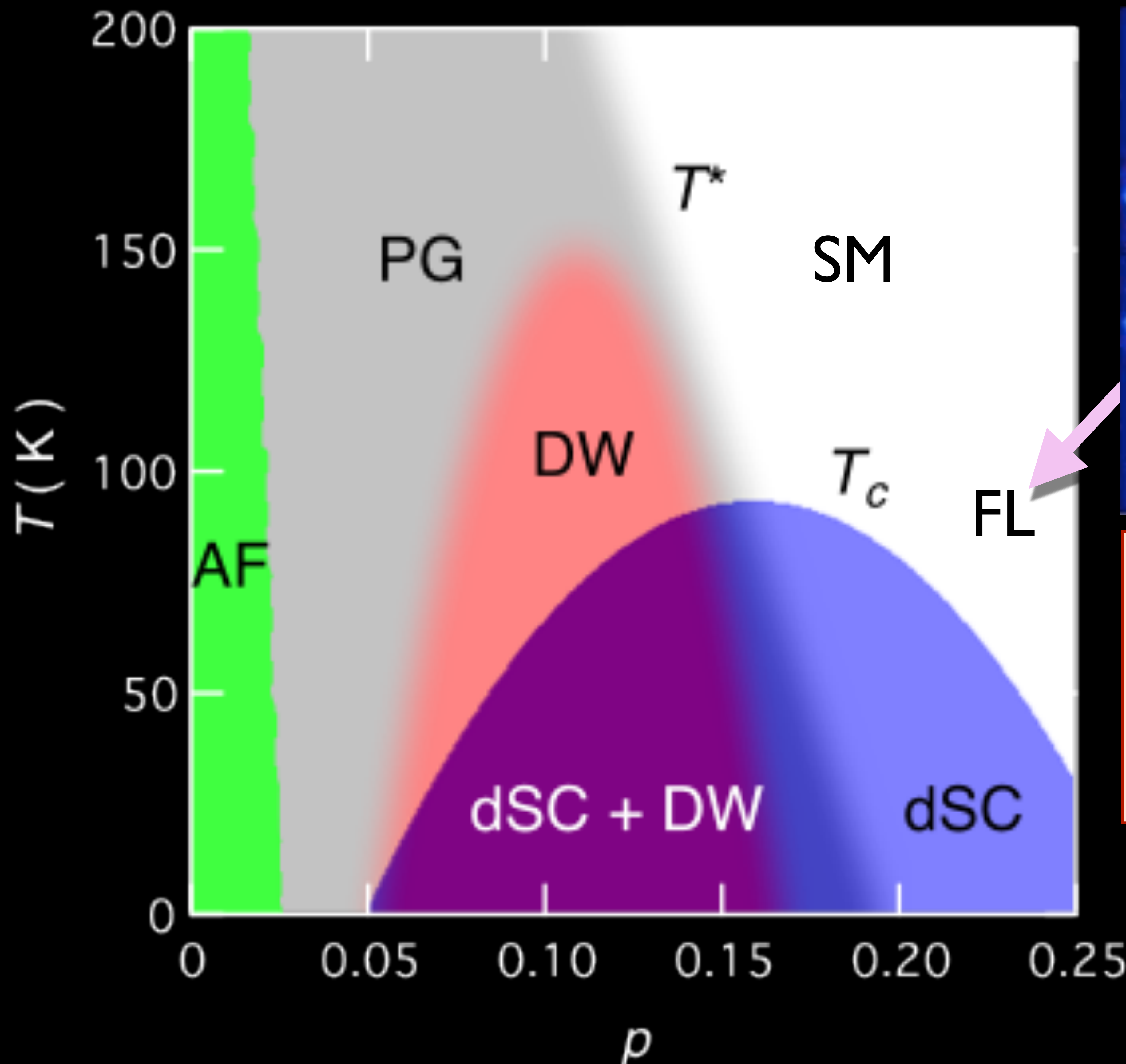
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

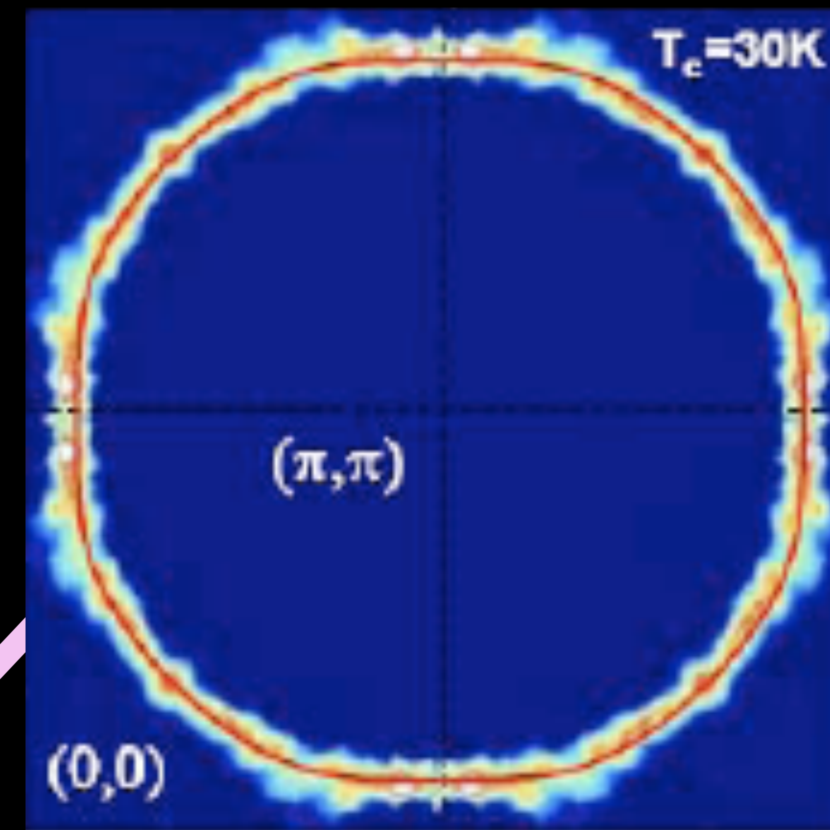




“Fermi arcs” at low p

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)





Conventional metal

Area enclosed by Fermi surface =1+p

SCIENCE VOL 323 30 JANUARY 2009
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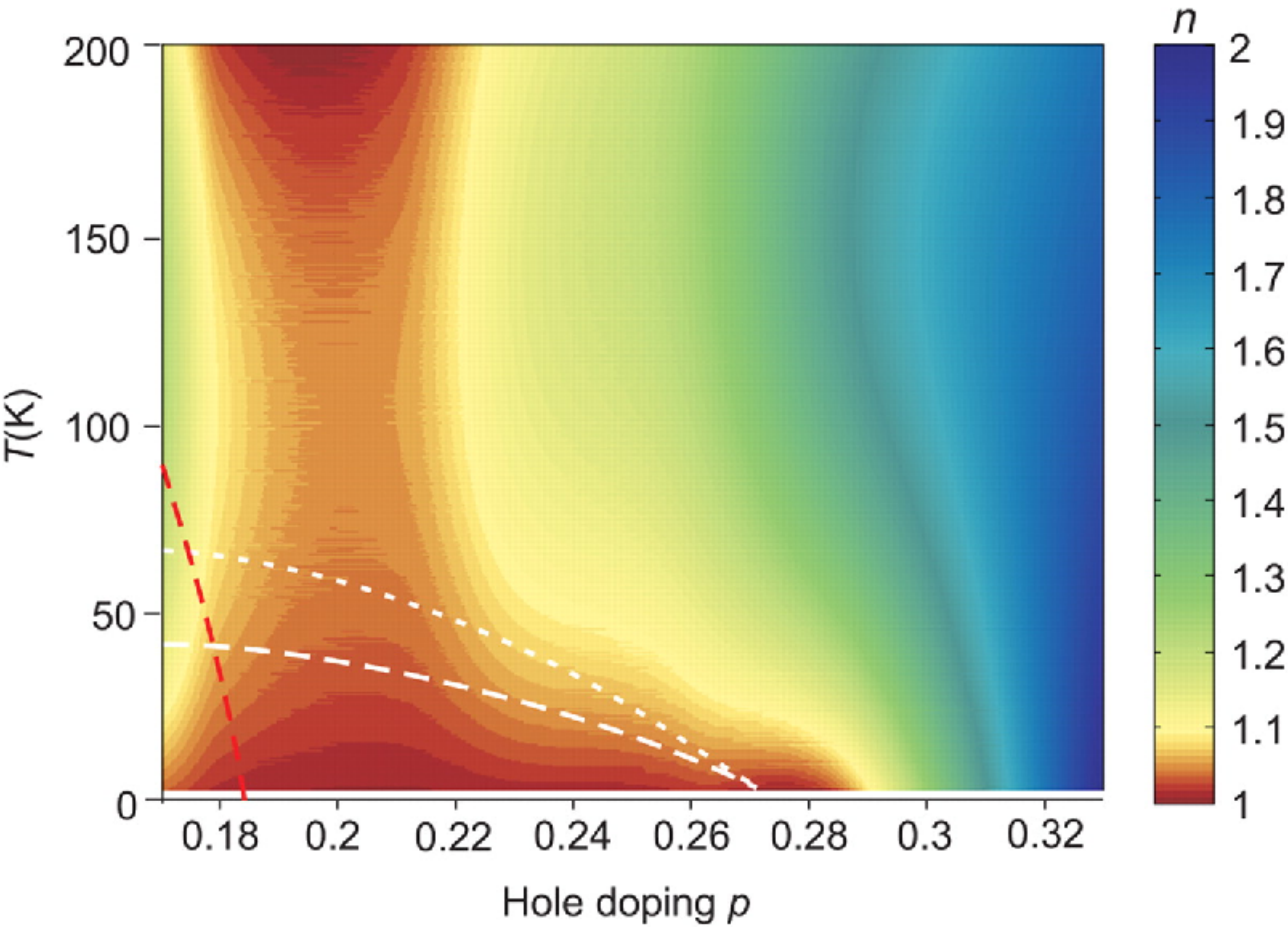
Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4]* H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†



Universal *T*-linear resistivity and Planckian limit in overdoped cuprates

arXiv:1805.02512

A. Legros[1,2], S. Benhabib[3], W. Tabis[3,4], F. Laliberté[1], M. Dion[1], M. Lizaire[1],

B. Vignolle[3], D. Vignolles[3], H. Raffy[5], Z. Z. Li[5], P. Auban-Senzier[5],

N. Doiron-Leyraud[1], P. Fournier[1,6], D. Colson[2], L. Taillefer[1,6], and C. Proust[3,6]

From the resistivity, they determined the value of the number α defined by

$$\rho(T) = \rho_0 + \alpha \frac{h}{2e^2} \left(\frac{T}{T_F} \right)$$

where $T_F = (\pi\hbar^2/k_B)(n/m^*)$ and m^* is determined from the specific heat. This expression is obtained from the Drude form $\rho = m^*/(ne^2\tau)$ and $\hbar/\tau = \alpha\, k_B T$.

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Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Slope of *T*-linear resistivity vs Planckian limit in seven materials.

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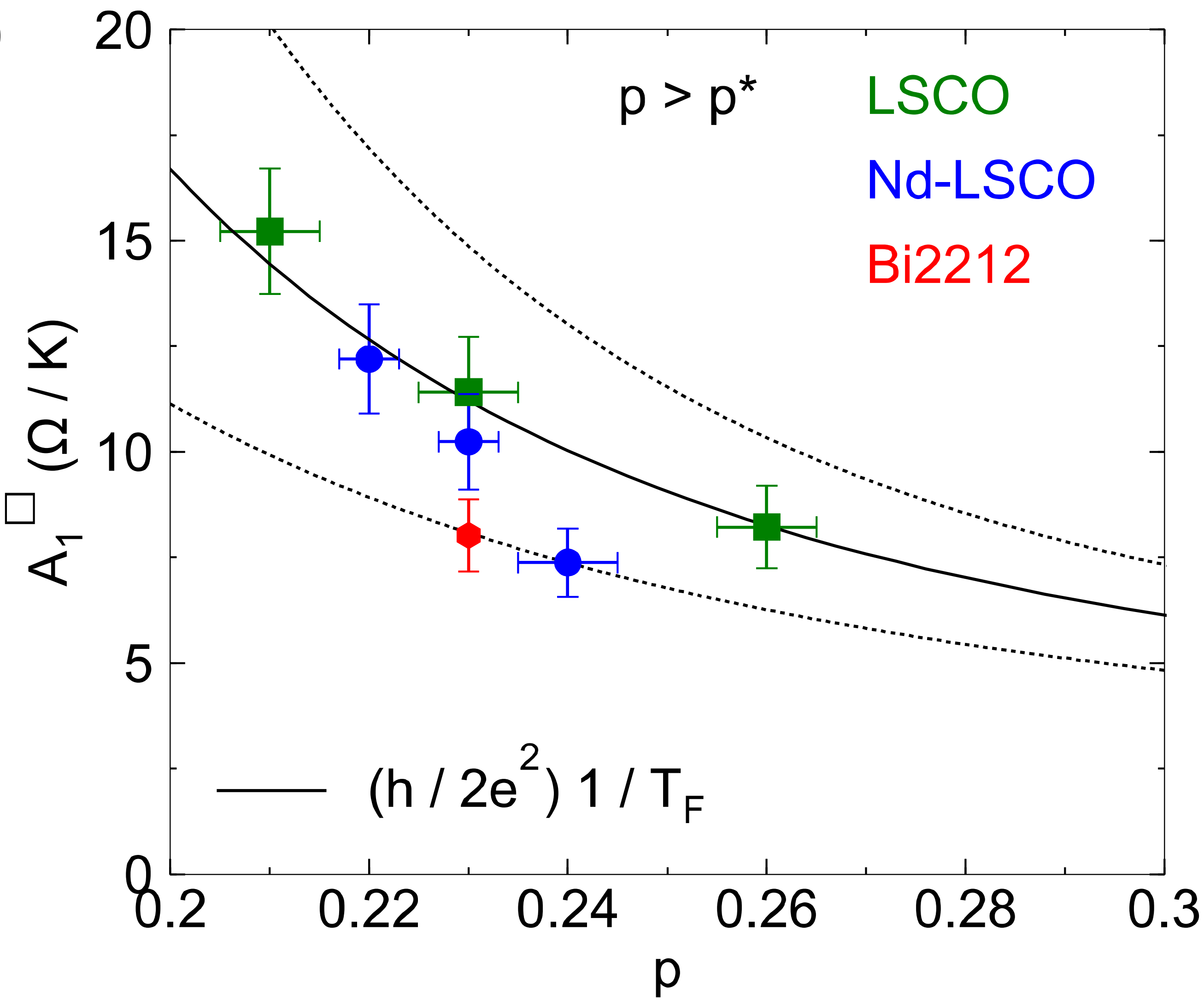

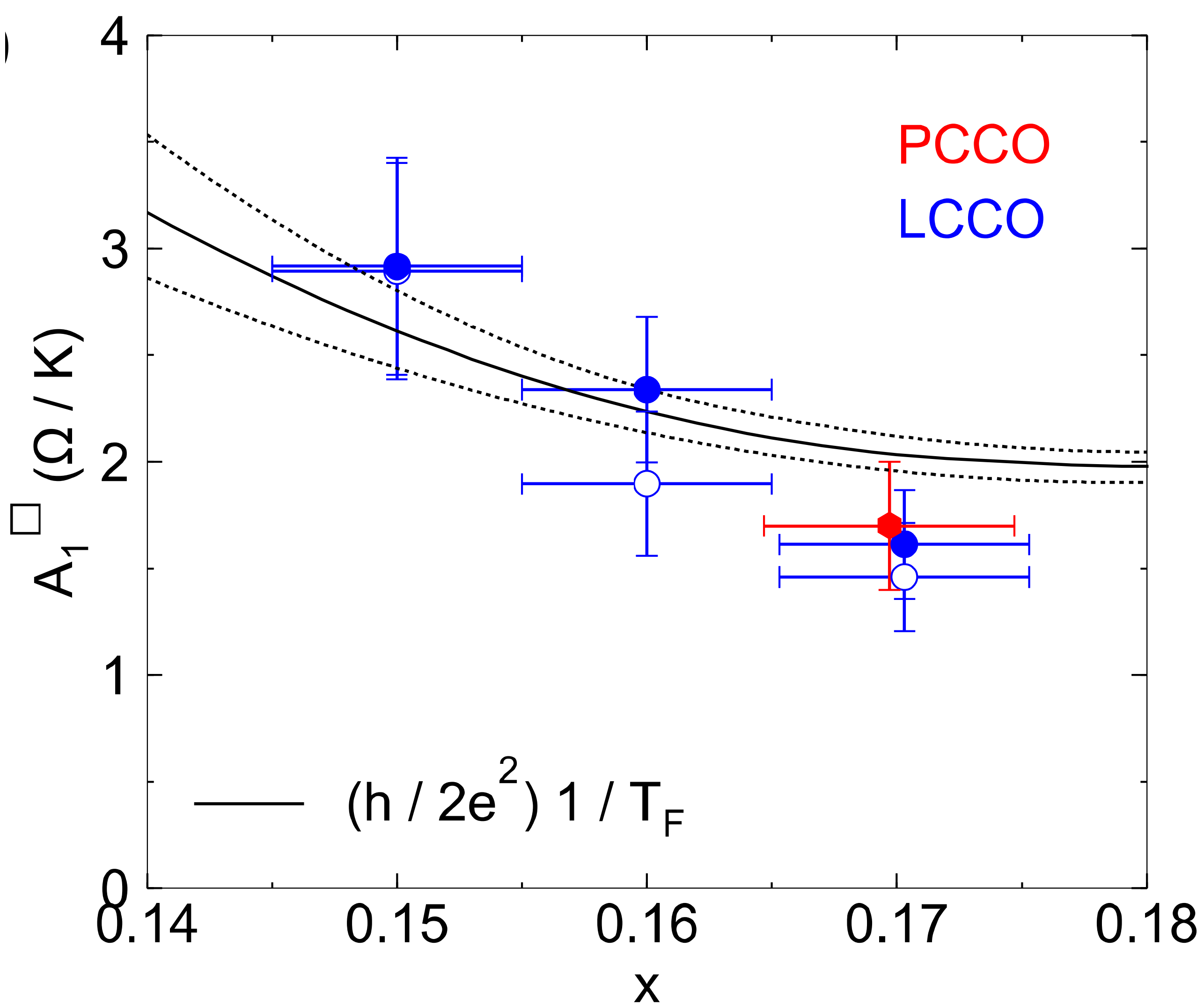

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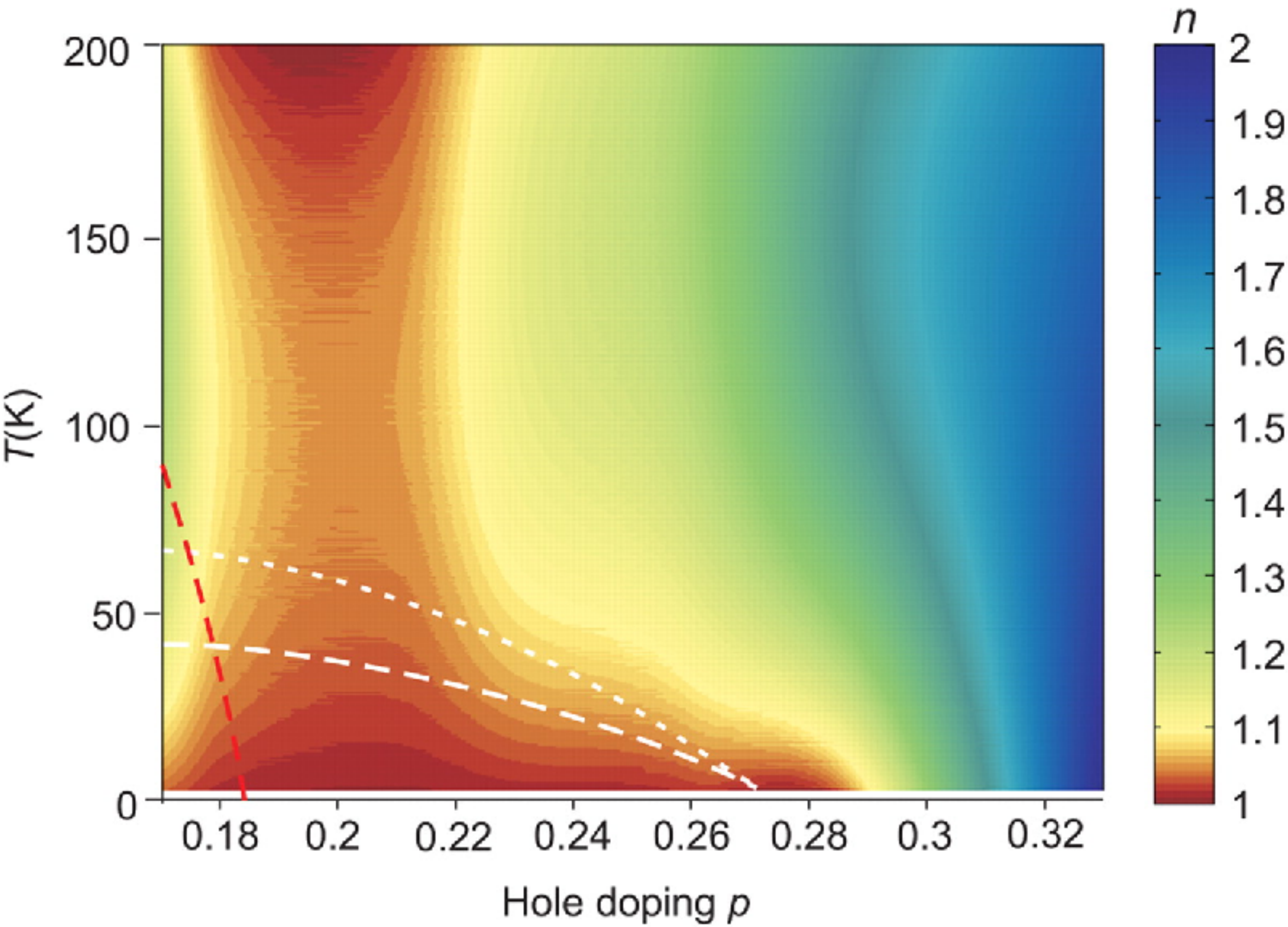


1. Metal with quasiparticles
   Random matrix model of a `quantum dot'

2. Metal without quasiparticles
   SYK model of a `quantum dot'

3. Lattice models of SYK islands
   Theory of a strange metal

4. Z_2 Fractionalization in a SYK *t-J* model

5. SYK U(1) gauge theory

Orthogonal metals

Fractionalize the electron $c_{i\alpha}$, $\alpha =\uparrow,\downarrow$ into an "orthogonal fermion" $f_{i\alpha}$ and an Ising spin $\sigma_i^z = \pm 1$:

$$c_{i\alpha} = \sigma_i^z \, f_{i\alpha}$$

This introduces a $\mathbb{Z}_2$ gauge invariance

$$\sigma_i^z \rightarrow \eta_i \sigma_i^z \quad , \quad f_{i\alpha} \rightarrow \eta_i f_{i\alpha}$$

The orthogonal fermion, f_α, carries both the spin and charge of the electron.
The Ising matter field, σ^z, is 'dark matter' carrying only energy, and a $\mathbb{Z}_2$ gauge charge.

R. Nandkishore, M. Metlitski, and T. Senthil, Phys. Rev. B 86, 045128 (2012)

Orthogonal metals

Fractionalize the electron $c_{ip\alpha}$, on sites $i = 1 \ldots N$, with spin $\alpha = 1 \ldots M$ and orbital index $p = 1 \ldots M'$ into an "orthogonal fermion" $f_{i\alpha}$ and a real scalar ϕ_{ip}:

$$c_{ip\alpha} = \phi_{ip} f_{i\alpha}$$

This introduces a $\mathbb{Z}_2$ gauge invariance

$$\phi_{ip} \rightarrow \eta_i \phi_{ip} \quad , \quad f_{i\alpha} \rightarrow \eta_i f_{i\alpha}$$

The orthogonal fermion $f_{i\alpha}$ carries both the spin and charge of the electron.
The scalar field, ϕ_{ip}, is 'dark matter' carrying only energy, and a $\mathbb{Z}_2$ gauge charge.

A solvable model

We examine the t-J model:

$$\mathcal{L} = \frac{1}{2g}\sum_{i,p}(\partial_\tau \phi_{ip})^2 + \sum_{i,\alpha} f_{i\alpha}^\dagger \left(\frac{\partial}{\partial\tau} - \mu\right) f_{i\alpha} + \frac{1}{\sqrt{NM}}\sum_{i,j,p,\alpha} t_{ij}\phi_{ip}\phi_{jp} f_{i\alpha}^\dagger f_{j\alpha} + \frac{1}{\sqrt{NM}}\sum_{i>j,\alpha\beta} J_{ij} f_{i\alpha}^\dagger f_{i\beta} f_{j\beta}^\dagger f_{j\alpha}\,,$$

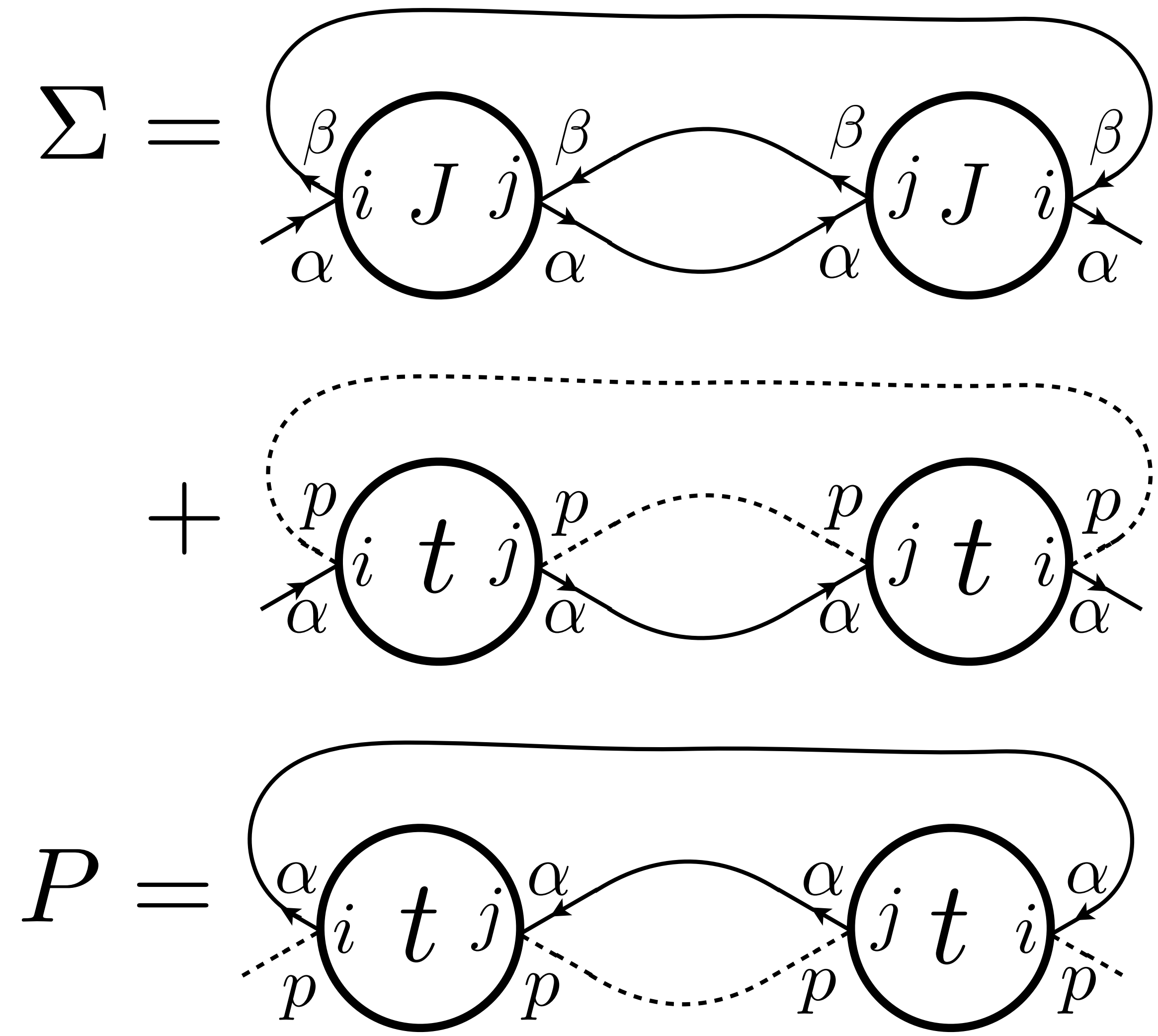
with the scalar field obeying the fixed length constraint

$$\sum_{p=1}^{M'} \phi_{ip}^2 = M'\,.$$

With t_{ij} and J_{ij} independent random numbers with zero mean, $\mathcal{L}$ is solvable in the limit of large number of sites, N, followed by the limit of large M and M' at fixed

$$k \equiv \frac{M'}{M}\,.$$

A solvable model



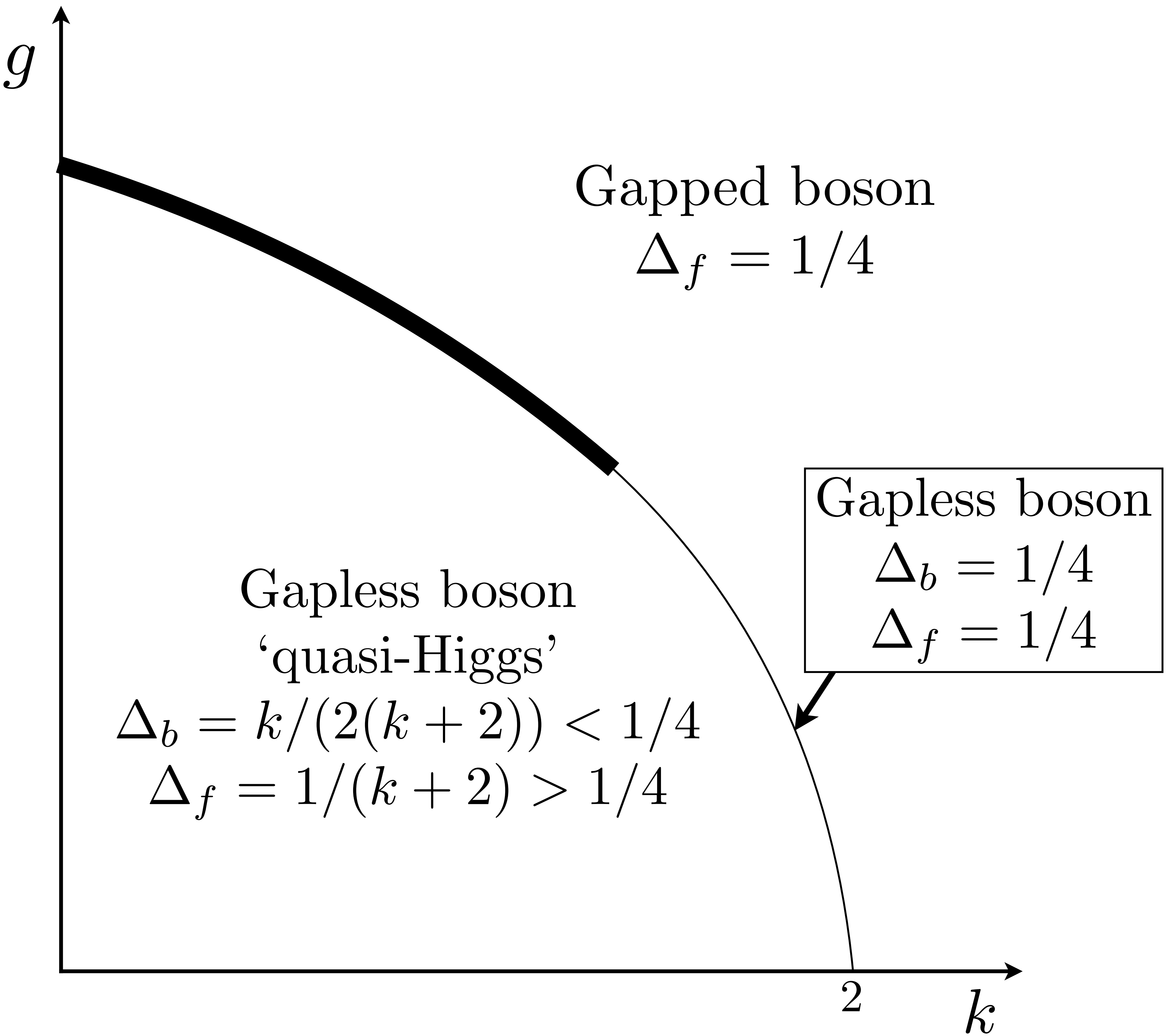
Equations for the fermion Green's function G and the boson Green's function χ:

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau) + k\,\tilde{t}^{\,2} G(\tau)\chi^2(\tau)$$

$$\chi(i\omega_n) = \frac{1}{\omega_n^2 + \chi_0^{-1} - P(i\omega_n) + P(i\omega_n = 0)} \quad , \quad P(\tau) = -2\,\tilde{t}^{\,2} G(\tau) G(-\tau) \chi(\tau)$$

where χ_0^{-1} is determined by solving $\chi(\tau = 0) = 1/g$, and $\tilde{t} = tJ$.

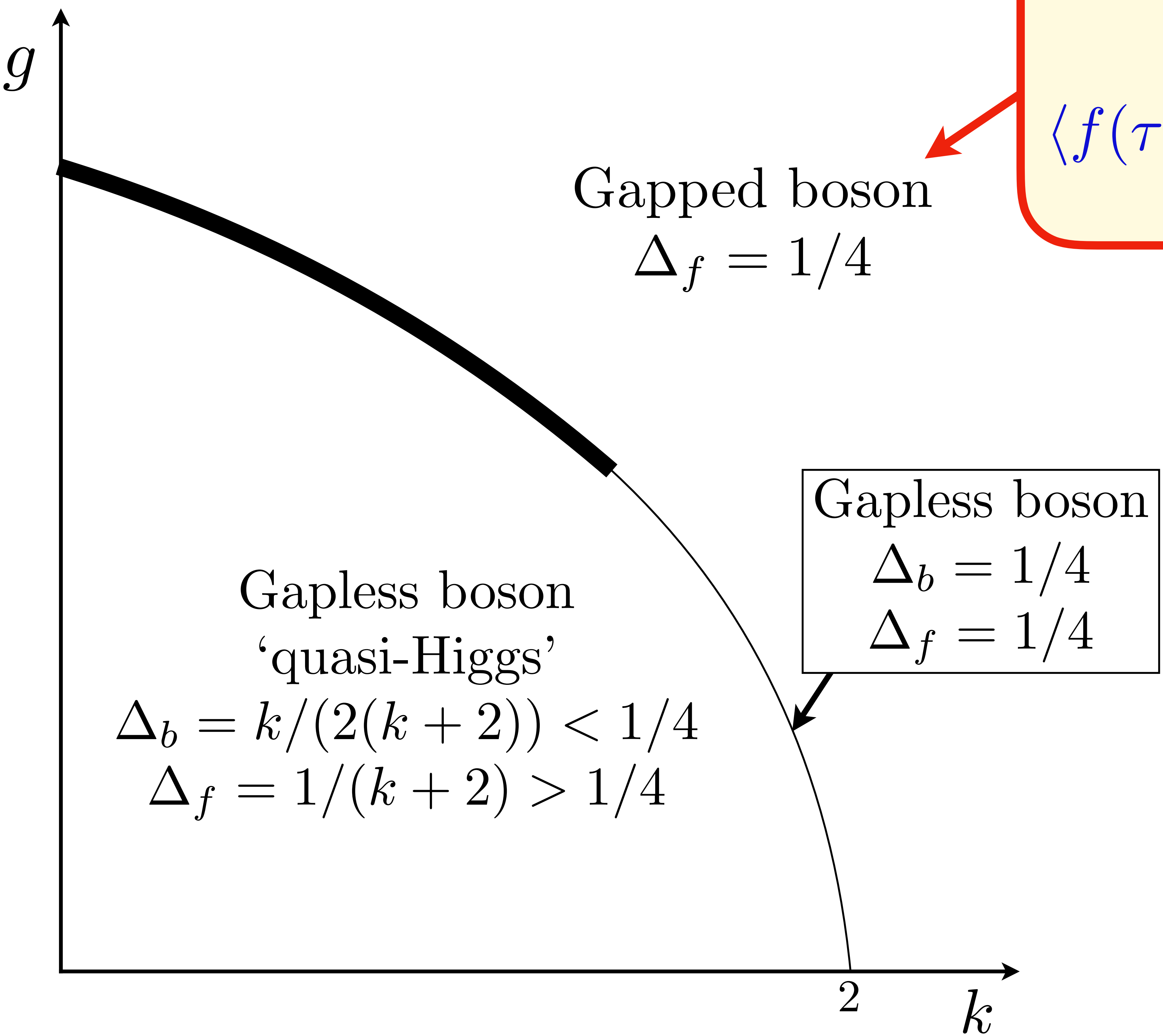
A solvable model



A solvable model

$$\langle \phi(\tau)\phi(0)\rangle \sim \frac{e^{-m|\tau|}}{\sqrt{\tau}}$$

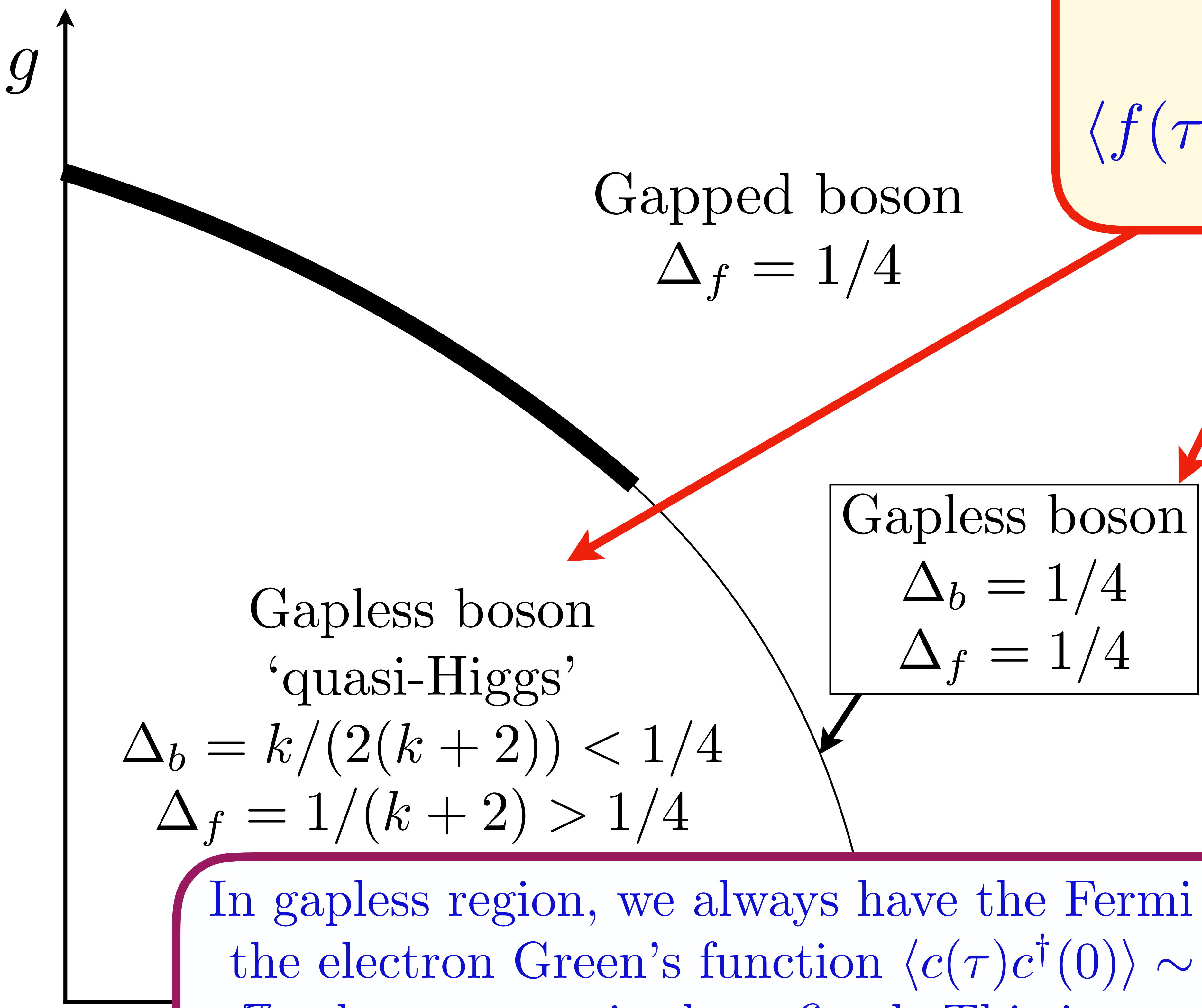
$$\langle f(\tau) f^\dagger(0)\rangle \sim \frac{\mathrm{sgn}(\tau)}{|\tau|^{2\Delta_f}}$$



A solvable model

$$\langle \phi(\tau)\phi(0)\rangle \sim \frac{1}{|\tau|^{2\Delta_b}}$$

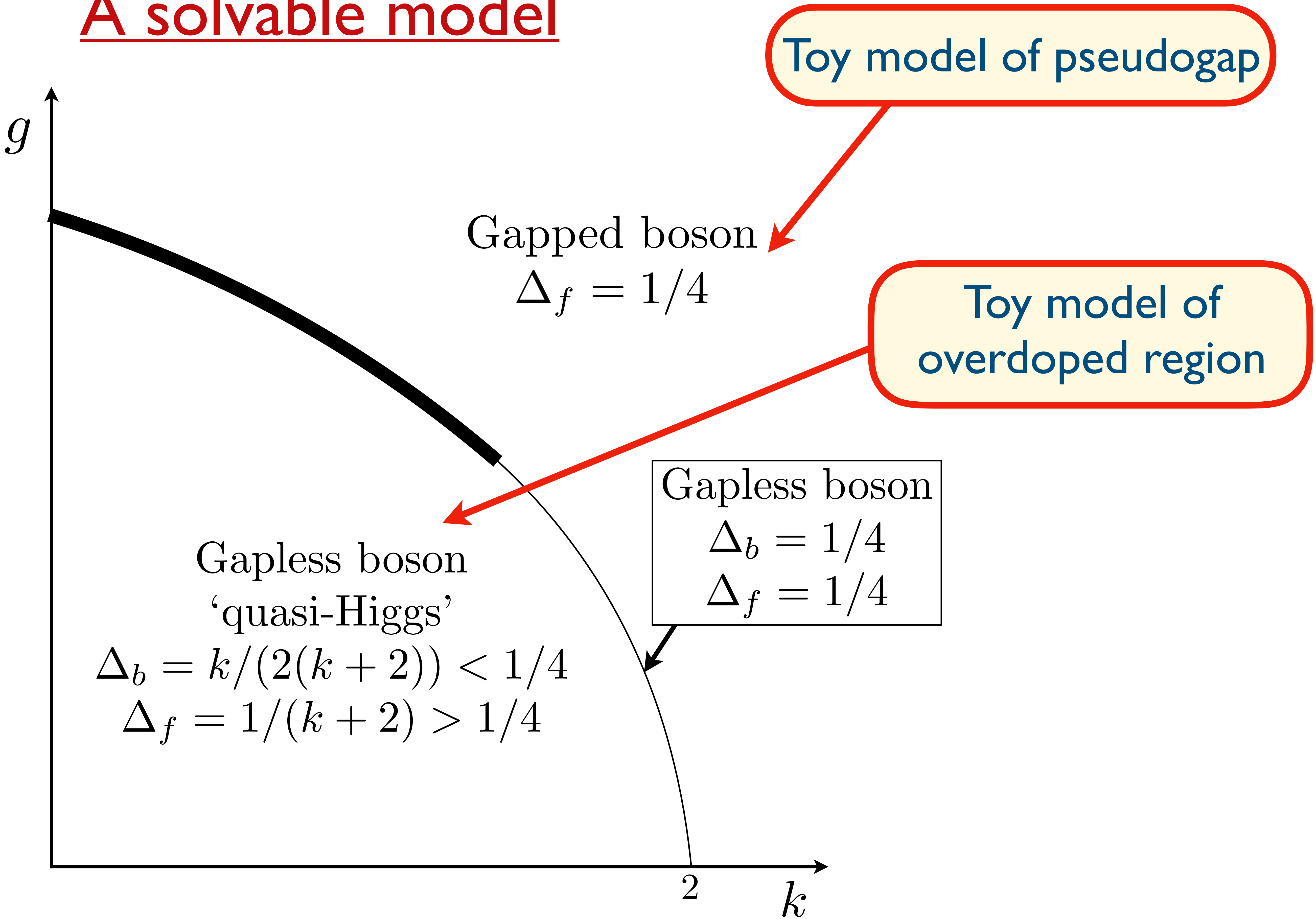
$$\langle f(\tau) f^\dagger(0)\rangle \sim \frac{\text{sgn}(\tau)}{|\tau|^{2\Delta_f}}$$



$$\Delta_b + \Delta_f = 1/2$$

In gapless region, we always have the Fermi liquid form for the electron Green's function $\langle c(\tau)c^\dagger(0)\rangle \sim 1/\tau$, although $\mathbb{Z}_2$ charges remain deconfined. This is a consequences of the fixed point with a non-zero t term in the t-J model.

A solvable model



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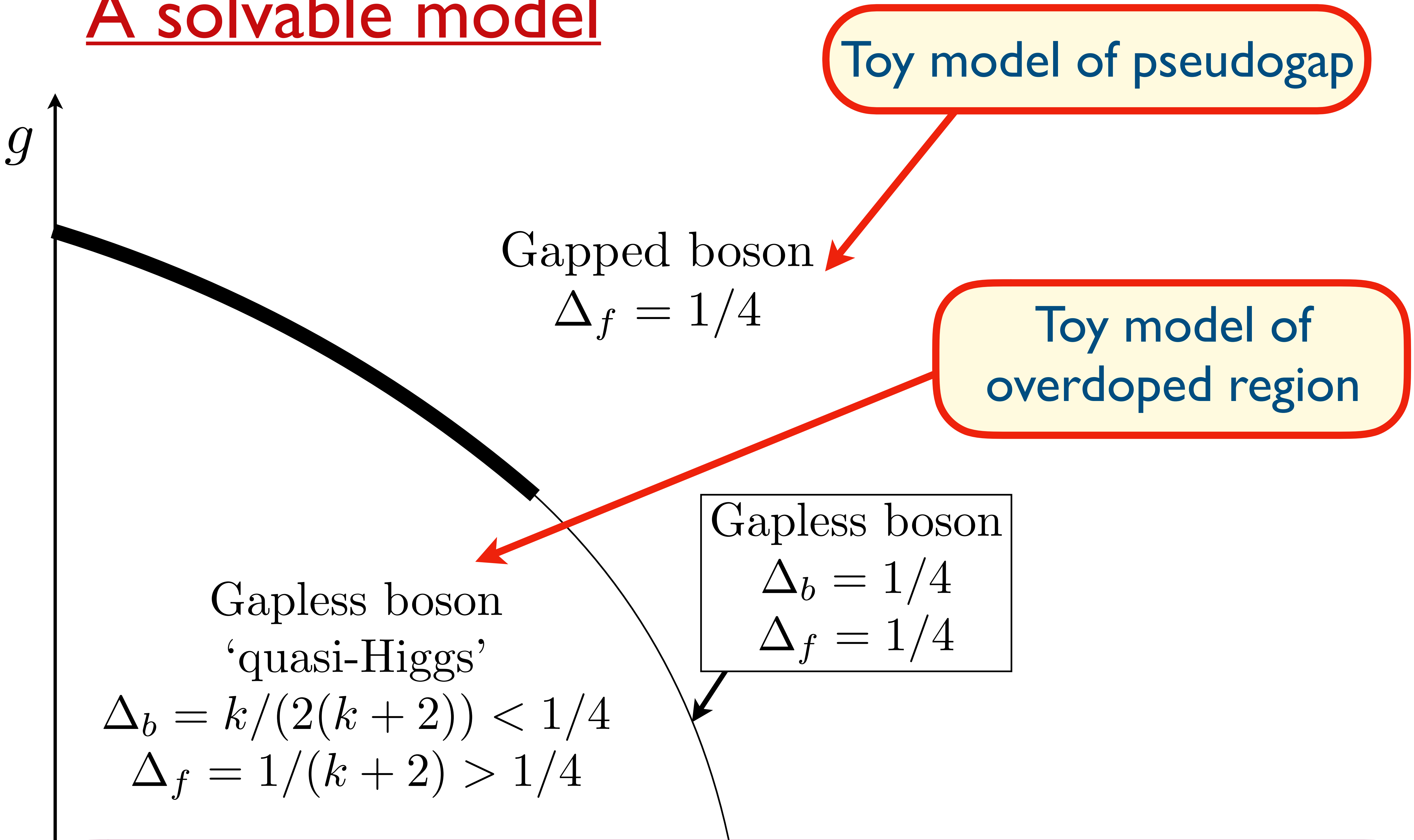
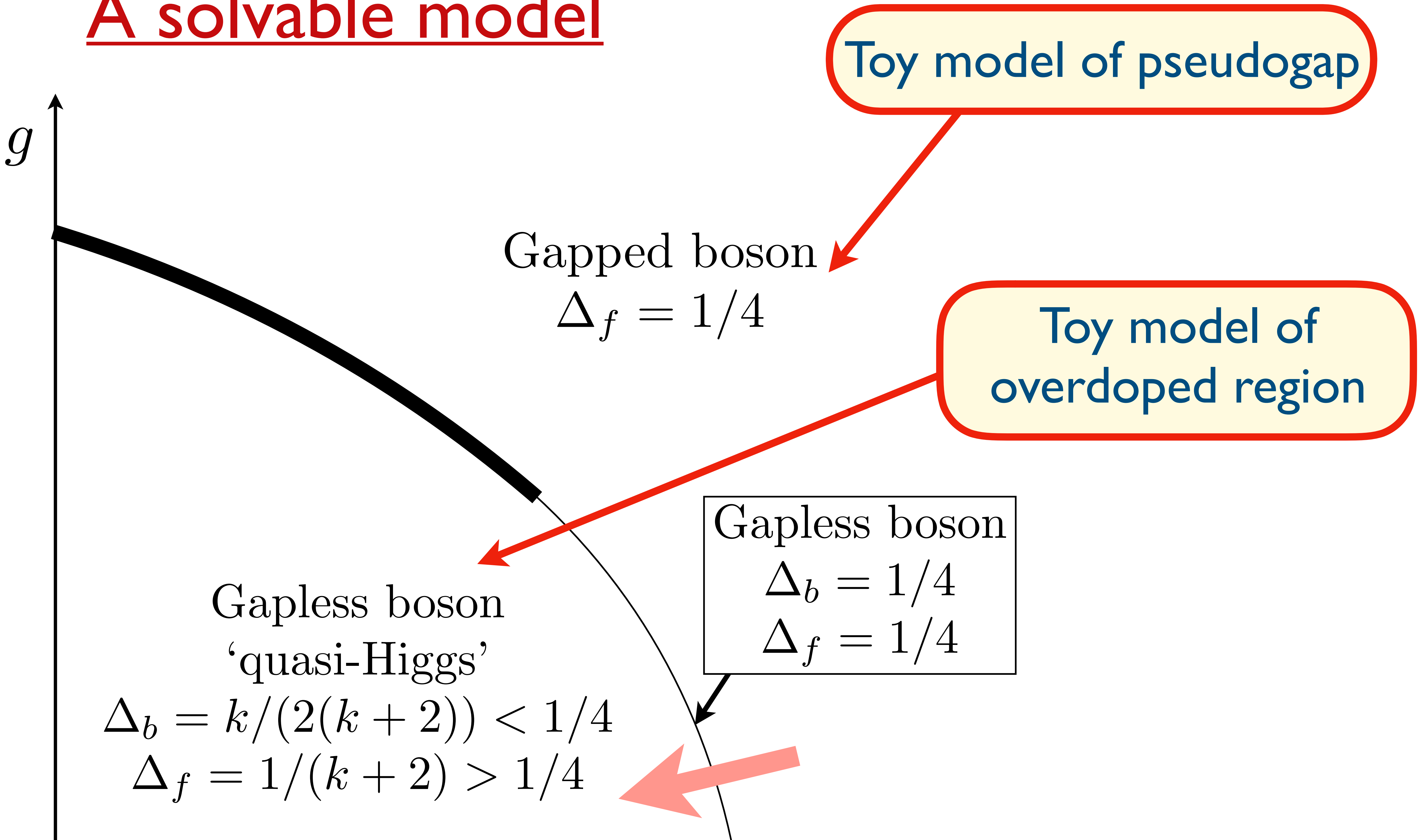


In the overdoped region, Fermi liquid electron spectral function, with anomalies in other properties, match recent observations in cuprates (Hussey, Bozovic, Armitage, Taillefer...)

A solvable model



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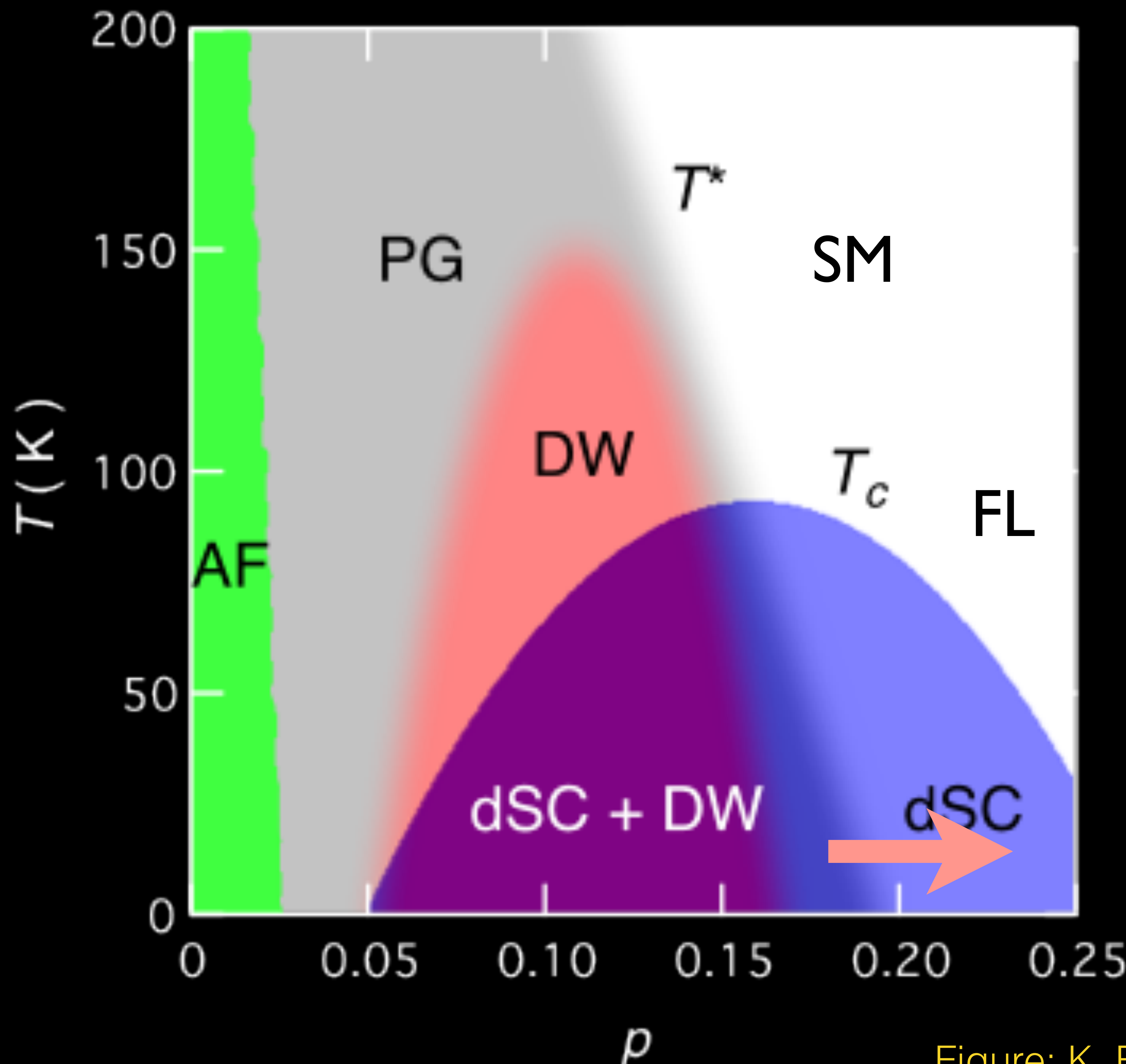

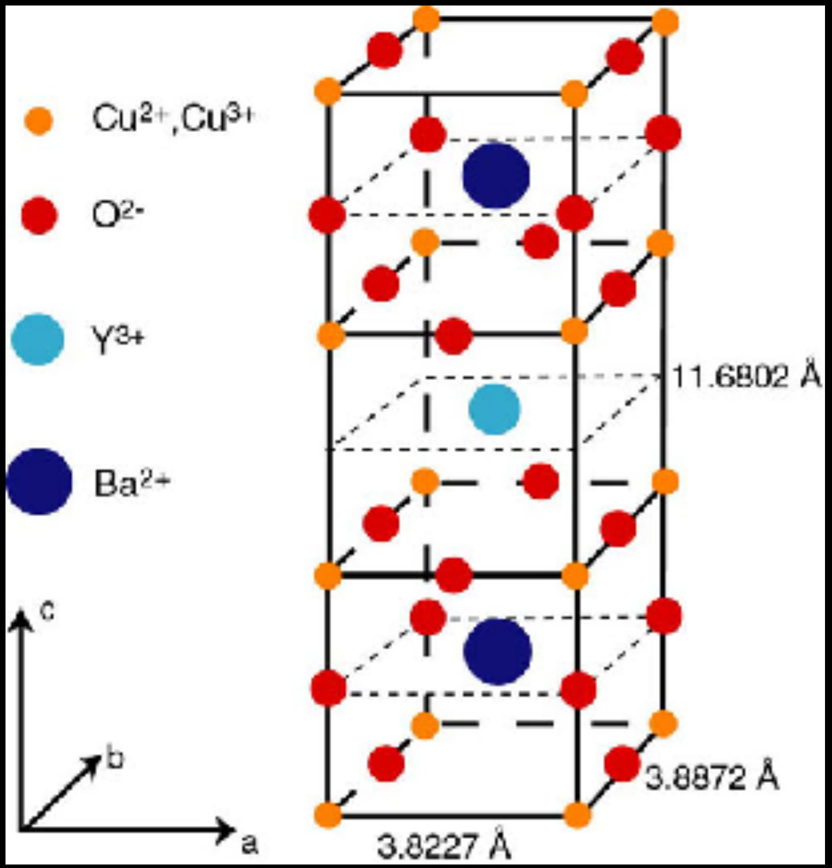

$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

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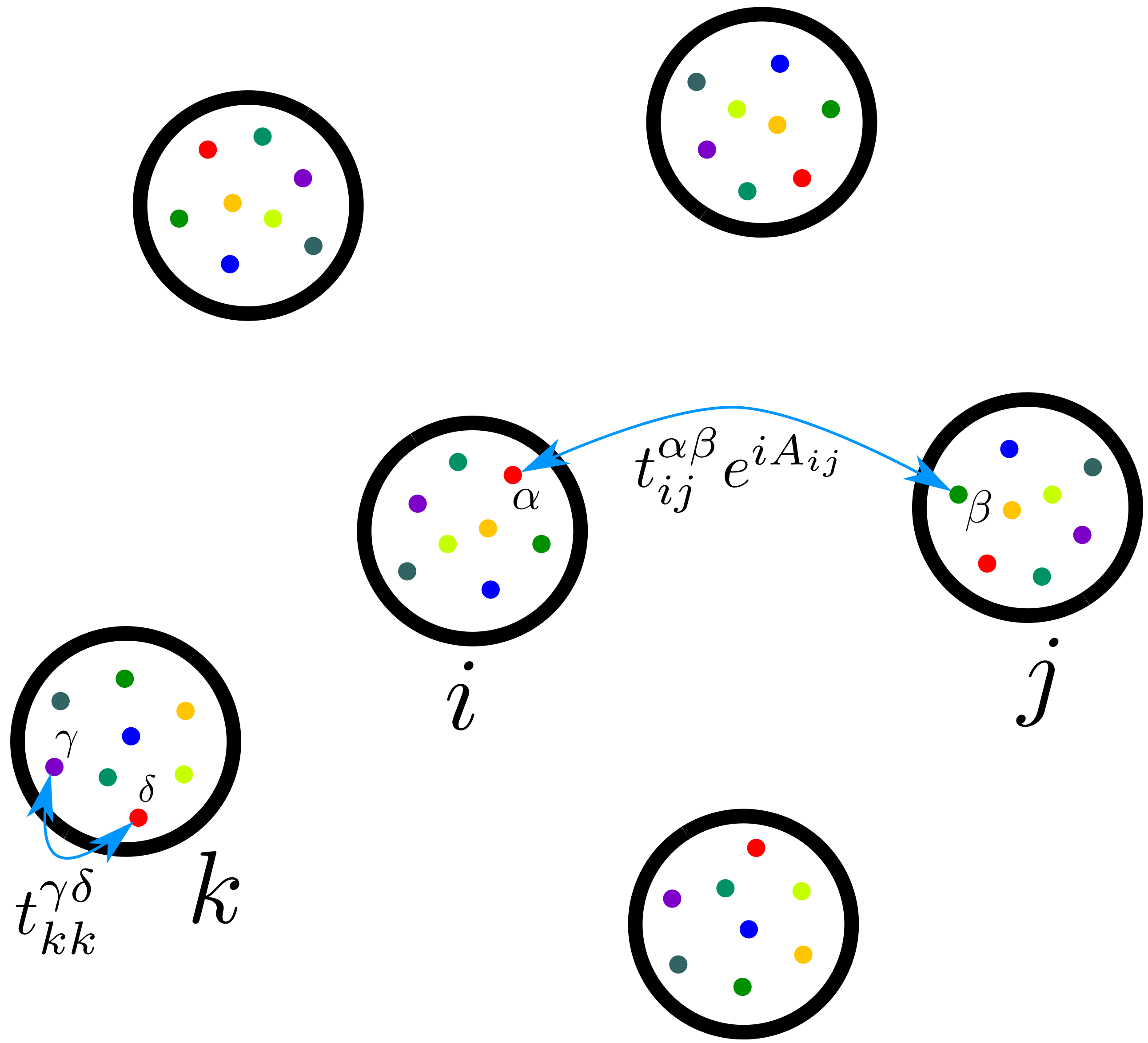
5. SYK U(1) gauge theory

Aavishkar Patel

Fermions with random hopping coupled to a fluctuating U(1) gauge field



$$H = -\frac{1}{(MN)^{1/2}}\sum_{ij=1}^{N}\sum_{\alpha\beta=1}^{M}\left[t_{ij}^{\alpha\beta}e^{iA_{ij}}f_{i\alpha}^{\dagger}f_{j\beta} + (MN)^{1/2}\mu\delta_{ij}^{\alpha\beta}f_{i\alpha}^{\dagger}f_{i\alpha}\right]$$

$$\ll t_{ij}^{\alpha\beta}t_{ji}^{\beta\alpha}\gg = \ll |t_{ij}^{\alpha\beta}|^2\gg = t^2, \quad A_{ji} = -A_{ij}.$$

Fermions with random hopping coupled to a fluctuating U(1) gauge field

$$\Sigma(i\omega_n) = t^2 G(i\omega_n) + t^2 T \sum_{\Omega_m \neq 0} \frac{G(i\omega_n + i\Omega_m) - G(i\omega_n)}{\Pi(i\Omega_m) - \Pi(i\Omega_m = 0)},$$

$$\Pi(i\Omega_m) = t^2 T \frac{M}{N} \sum_{\omega_n} G(i\omega_n) G(i\omega_n + i\Omega_m), \quad G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)}.$$

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General low energy solution

$$G(\tau > 0) = -\frac{C(\mathcal{E})}{t^{1-x}\tau^{1-x}}, \quad G(\tau < 0) = \frac{C(\mathcal{E}) e^{-2\pi\mathcal{E}}}{t^{1-x}|\tau|^{1-x}}.$$

where $\mathcal{E}$ is a parameter universally related to the filling fraction ($\mathcal{E} = 0$ at half-filling). The exponent x is the solution to

$$\frac{(1/x - 2)(\cosh(2\pi\mathcal{E}) - \cos(\pi x))}{\tan(\pi x)\sin(\pi x)} = \frac{M}{N}.$$

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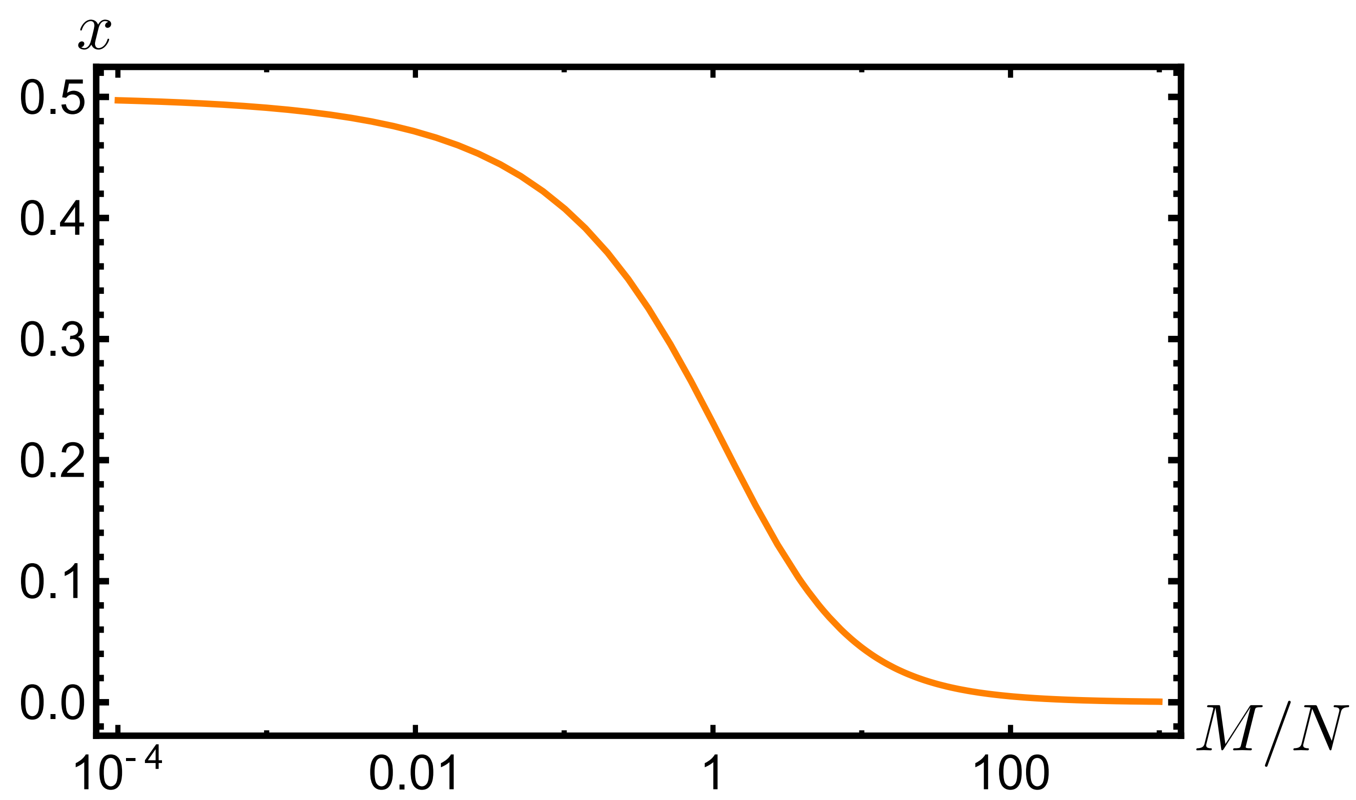

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$$\frac{(1/x - 2)(\cosh(2\pi\mathcal{E}) - \cos(\pi x))}{\tan(\pi x)\sin(\pi x)} = \frac{M}{N}.$$

Solvable models with fractionalization, variable fermion density, and disorder

1. Non-Fermi liquid regime of SYK models with linear-in-T resistivity

2. Z_2 Fractionalization in a SYK t-J model

Overdoped state described by state with fractionalization but with a Fermi liquid electron spectral function

3. SYK U(1) gauge theory