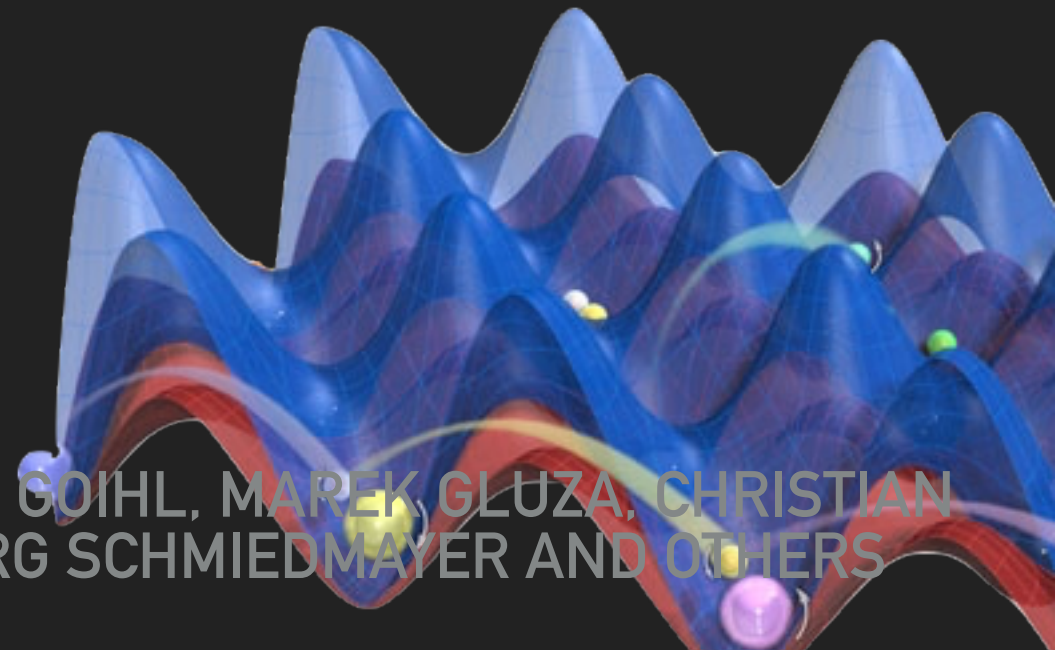


QUANTUM SIMULATORS

PROBING (NON)-INTEGRABLE DYNAMICS

JENS EISERT, FU BERLIN

JOINT WORK WITH HENRIK WILMING, INGO ROTH, MARCEL GOIHL, MAREK GLUZA, CHRISTIAN KRUMNOW, TERRY FARRELLY, THOMAS SCHWEIGLER, JOERG SCHMIEDMAYER AND OTHERS



**HOW DO (NON)-INTEGRABLE SYSTEMS
EQUILIBRATE AND THERMALIZE...?**

**...OR NOT, IN CASE OF
MANY-BODY LOCALIZATION?**



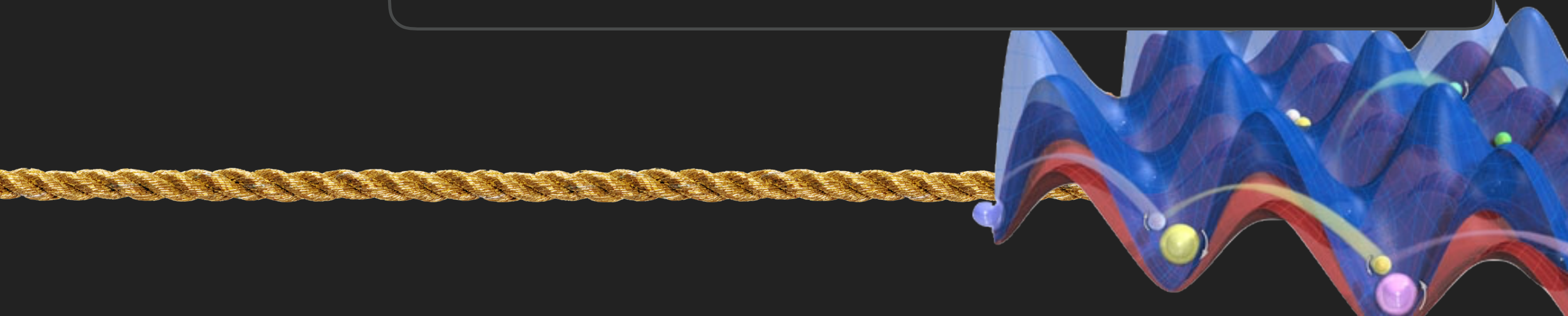
WHAT THIS TALK IS ABOUT

WHAT “WINDOWS” INTO PROBING THIS DO QUANTUM SIMULATORS PROVIDE?



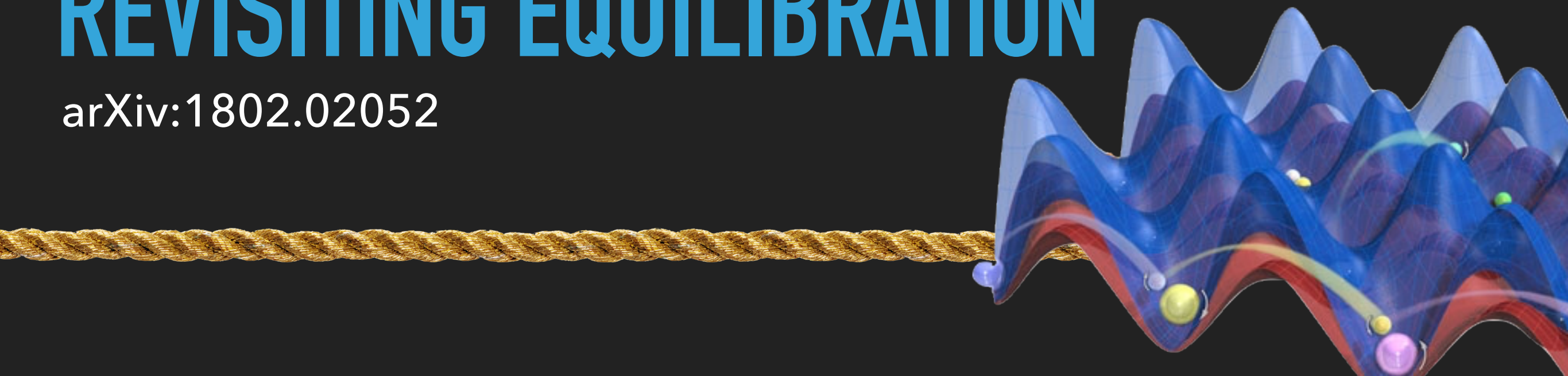
**CAN QUANTUM SIMULATORS SHOW A
“QUANTUM ADVANTAGE”, AND IF SO...**

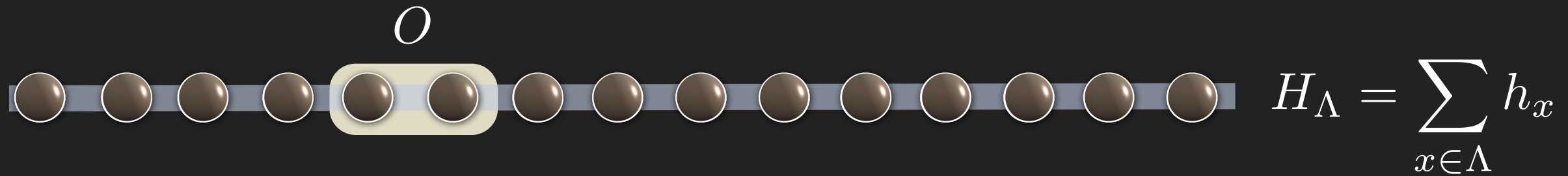
...HOW COULD WE EVER FIND OUT?



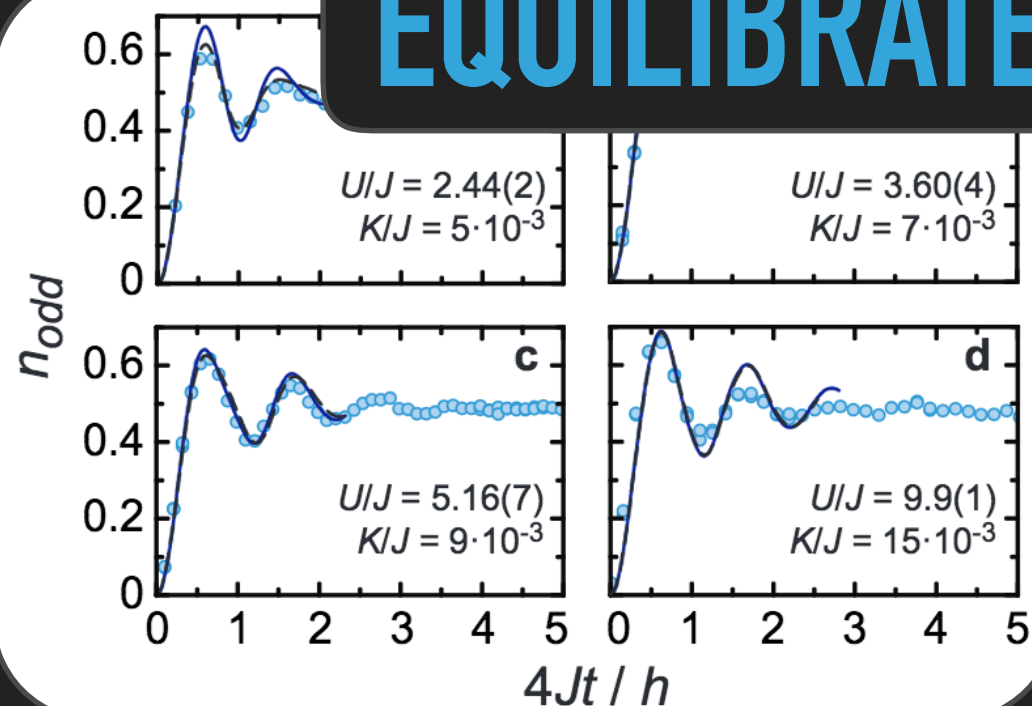
REVISITING EQUILIBRATION

arXiv:1802.02052

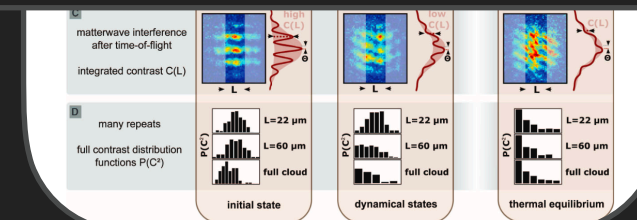




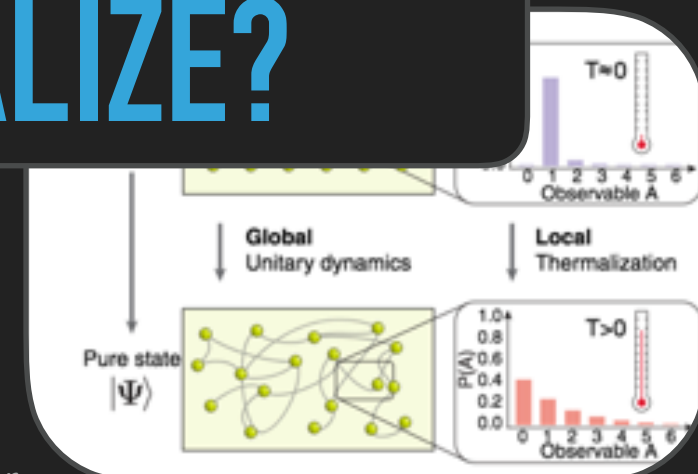
HOW DO NON-INTEGRABLE MODELS EQUILIBRATE AND THERMALIZE?



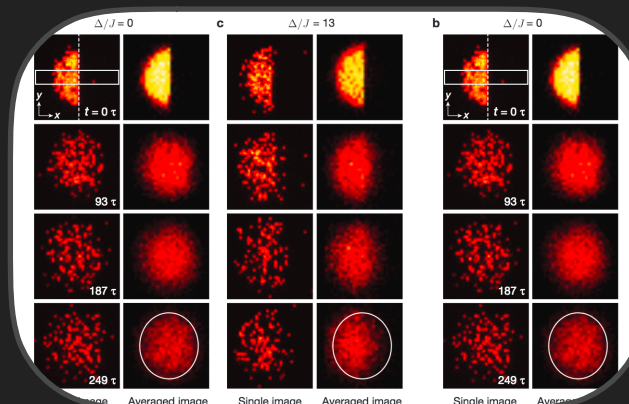
Trotzky, Chen, Flesch, McCulloch, Schollwöck, Eisert, Bloch, Nature Physics 8, 325 (2012)



Gring, Kuhnert, Langen, Kitagawa, Rauer, Schreitl, Mazets, Smith, Demler, Schmiedmayer, Science 337, 1318 (2012)

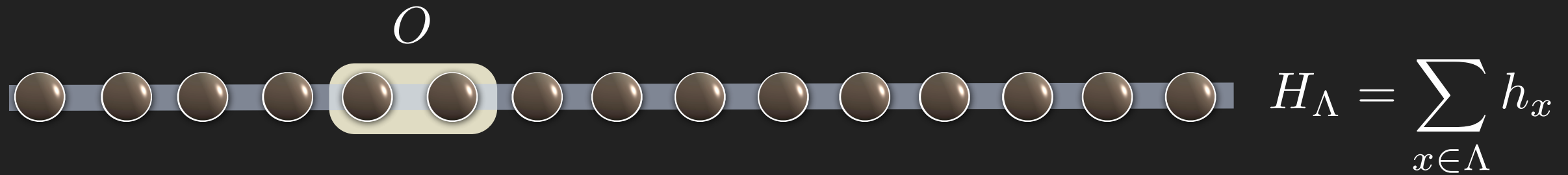


Kaufman, Tai, Lukin, Rispoli, Schittko, Preiss, Greiner, Science 353, 794 (2016)



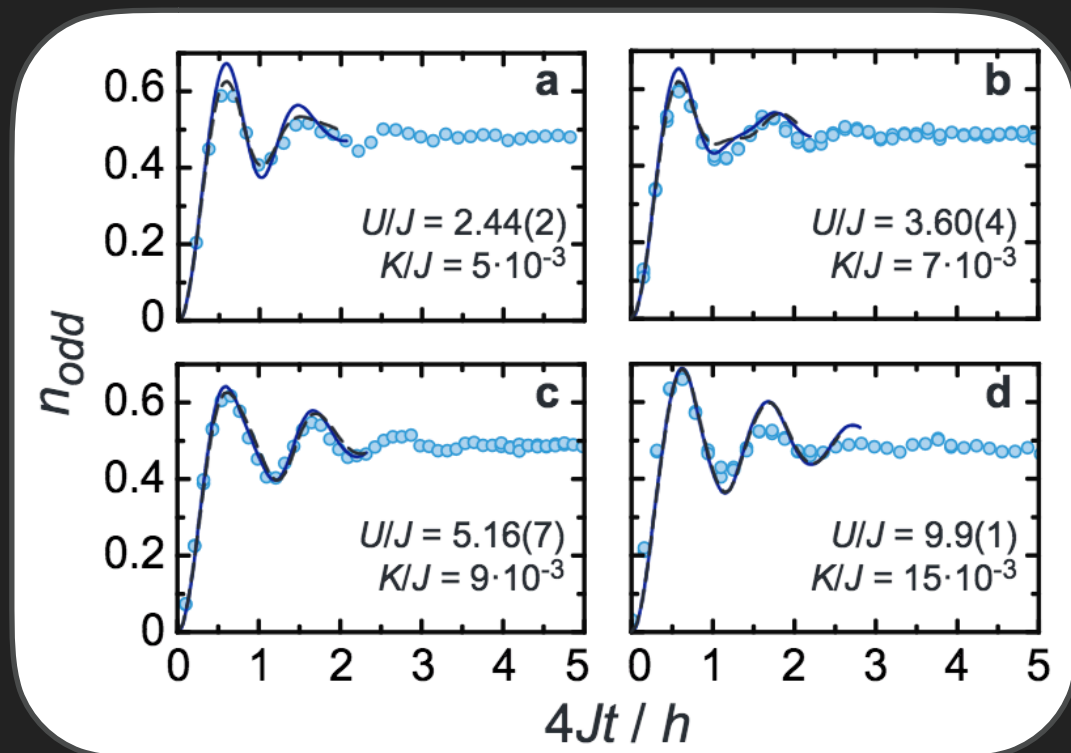
Choi, Hild, Zeiher, Schauß, Rubio-Abadal, Yefsah, Khemani, Huse, Gross, Science 352, 1547 (2016)

EQUILIBRATION

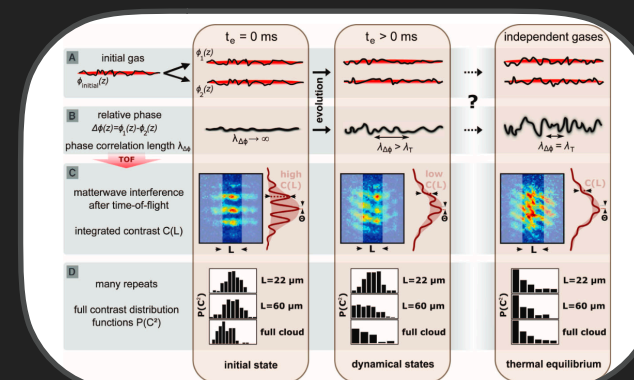


► Equilibration to **time averages** of local observables O

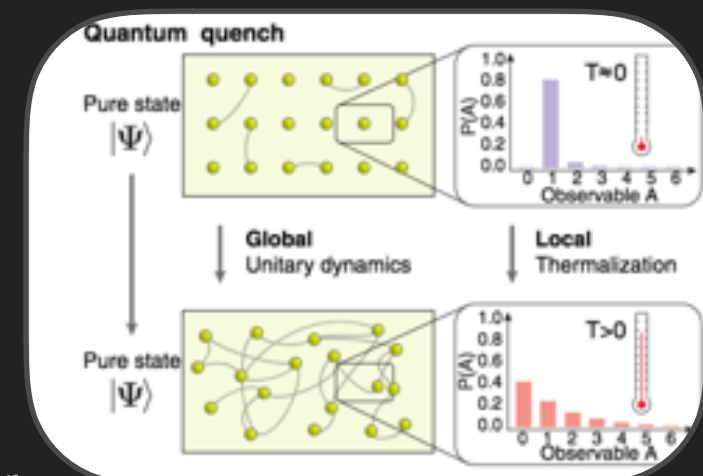
$$\bar{O} := \overline{\langle O(t) \rangle} = \lim_{T \rightarrow \infty} \int_0^T \text{tr}(\rho(t)O) = \text{tr}(\omega O)$$



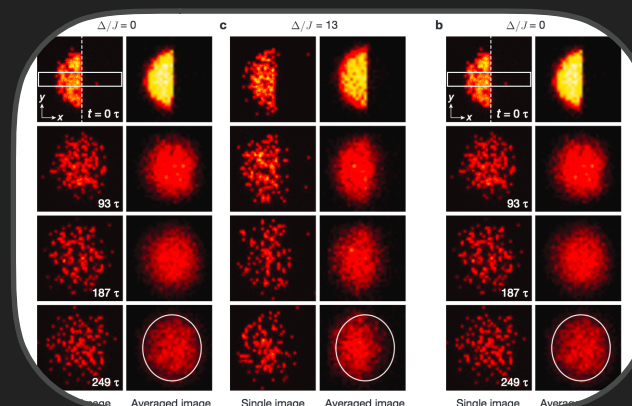
Trotzky, Chen, Flesch, McCulloch, Schollwöck, Eisert, Bloch, Nature Physics 8, 325 (2012)



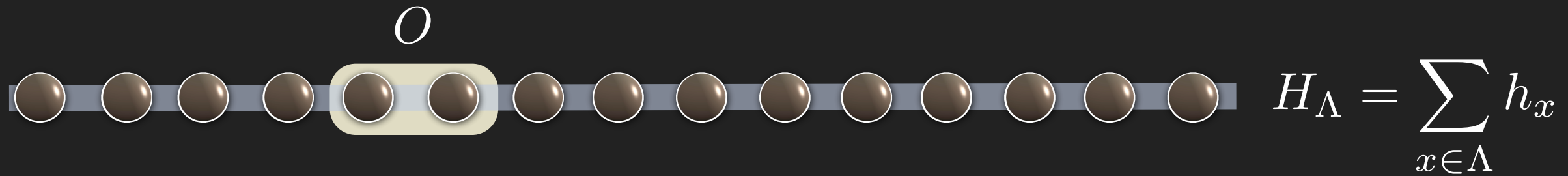
Gring, Kuhnert, Langen, Kitagawa, Rauer, Schreitl, Mazets, Smith, Demler, Schmiedmayer, Science 337, 1318 (2012)



Kaufman, Tai, Lukin, Rispoli, Schittko, Preiss, Greiner, Science 353, 794 (2016)



Choi, Hild, Zeiher, Schauß, Rubio-Abadal, Yefsah, Khemani, Huse, Gross, Science 352, 1547 (2016)

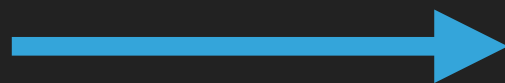


- Equilibration to **time averages** of local observables O

$$\overline{O} := \overline{\langle O(t) \rangle} = \lim_{T \rightarrow \infty} \int_0^T \text{tr}(\rho(t)O) = \text{tr}(\omega O)$$



- Invoke **eigenstate therm hypothesis (ETH)**

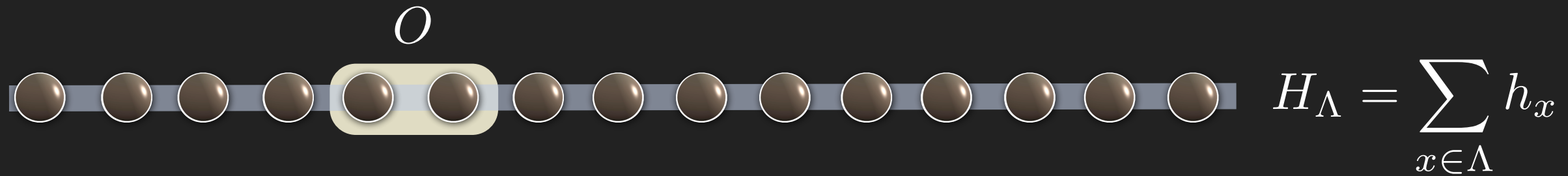


- **Thermalization:**
"Form its own heat bath"

$$\text{tr}_{\setminus A}(|e\rangle\langle e|) = |e\rangle\langle e|_A \sim \text{tr}_{\setminus A} \left(\frac{e^{-\beta H}}{Z} \right)$$

Deutsch, Phys Rev A 43, 2046 (1991)

Srednicki, Phys Rev E 50, 888 (1994)



- Equilibration to **time averages** of local observables O

$$\overline{O} := \overline{\langle O(t) \rangle} = \lim_{T \rightarrow \infty} \int_0^T \text{tr}(\rho(t)O) = \text{tr}(\omega O)$$

- Deviations from time average

$$\text{Var}(O, H, \rho) := (\overline{\langle O(t) \rangle} - \overline{O})^2 \leq \|O\|^2 e^{-S_2(\omega)}$$

in terms of Renyi entropy $S_\alpha(\rho) := \frac{1}{1-\alpha} \log(\text{tr}(\rho^\alpha))$

- Systems **equilibrate** if “effective dimension” $S_2(\omega)$ is large

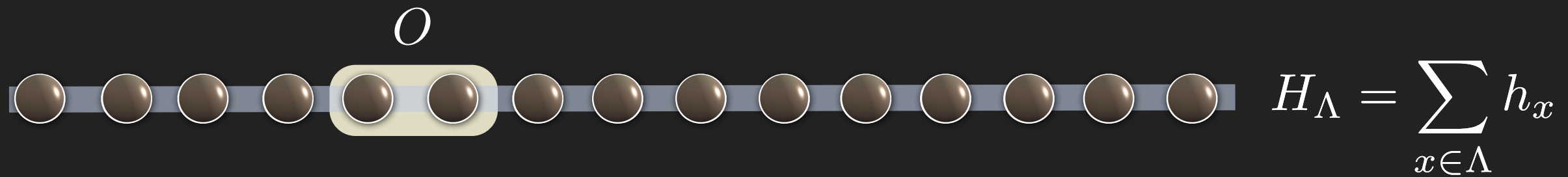
Reimann, Phys Rev Lett 101, 190403 (2008)

Linden, Popescu, Short, Winter, Phys Rev E 79, 61103 (2009)

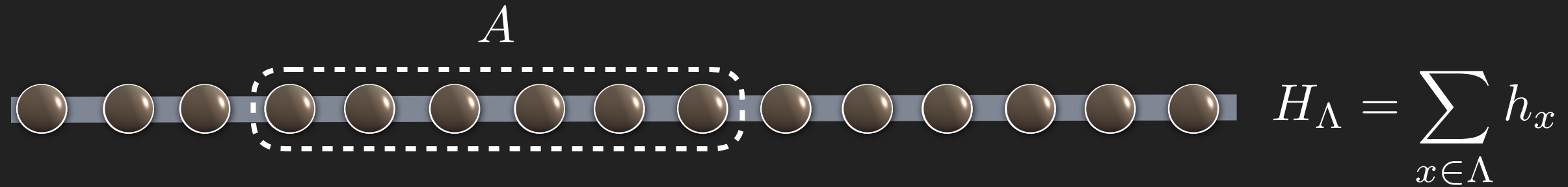
Reimann, Kastner, New J Phys 14, 43020 (2012)

Short, Farrelly, New J Phys 14, 013063 (2012)

Gogolin, Eisert, Rep Prog Phys 79, 56001 (2016)



**BUT WHEN IS THE EFFECTIVE DIMENSION
LARGE? A DIFFERENT TAKE...**



- ▶ Generic eigenvectors $\{|e\rangle\}$ of local Hamiltonians* are expected to satisfy a **volume law** for the **entanglement entropy**

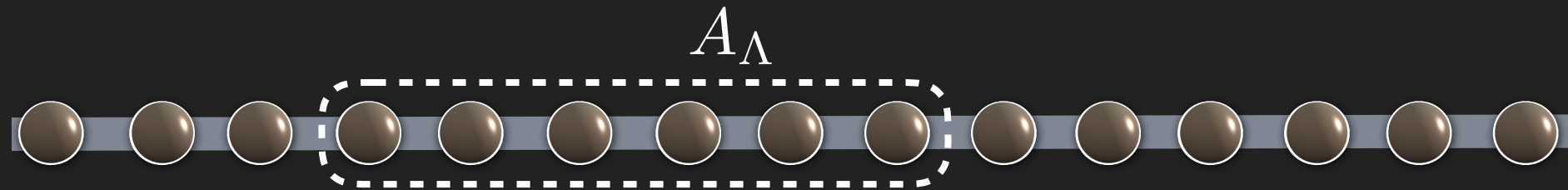
$$S_\alpha(\rho_A(e)) \sim |A|$$

▶ **FOR WHAT VALUES OF α IS THIS MEANINGFUL?**

- ▶ For $\alpha > 1$ only

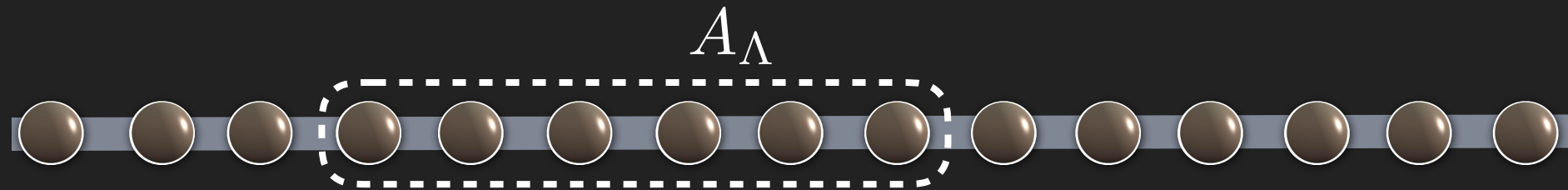
- ▶ $|\psi^\epsilon\rangle$ has overlap exponentially close to $1 - \epsilon$ with a product state
- ▶ $|\psi^\epsilon\rangle$ fulfills a volume law for the von-Neumann entropy
- ▶ All Renyi entropies for $\alpha > 1$ are upper bounded by a constant

$$\lim_{\Lambda \rightarrow \infty} \frac{1}{N} \log \dots = c$$



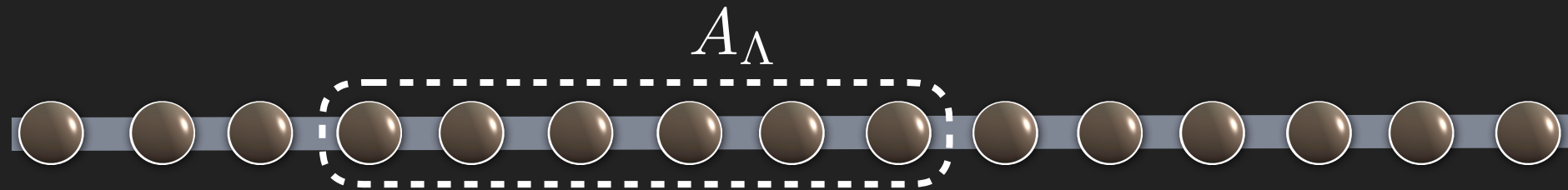
- ▶ **Entanglement ergodicity:** A system is EG, if there ex a positive function g , such that for energy eigenvectors $|e\rangle_\Lambda$ with energy density e the conditions hold
 - ▶ For every sufficiently large lattice Λ there exists a subsystem A_Λ such that the reduced state $\rho_{A_\Lambda}(e) = \text{tr}_{A_\Lambda^c}(|e\rangle\langle e|_\Lambda)$ fulfills

$$S_2(\rho_{A_\Lambda}(e)) \geq g(e)N$$
 - ▶ The function g is sufficiently well behaved
- ▶ *2-Renyi entropy* taken for convenience
- ▶ *Entanglement ergodicity* is stable under short evolution
- ▶ Is what *many-body localization* is **not** about

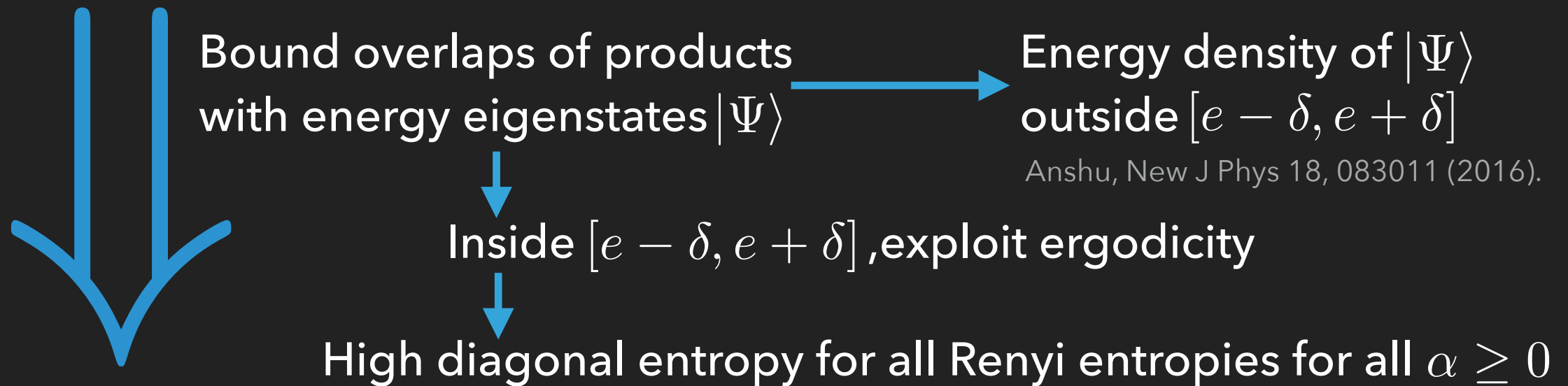


► **Entanglement ergodicity:** $S_2(\rho_{A_\Lambda}(e)) \geq g(e)N$

WHAT DOES THIS MEAN FOR EQUILIBRATION?



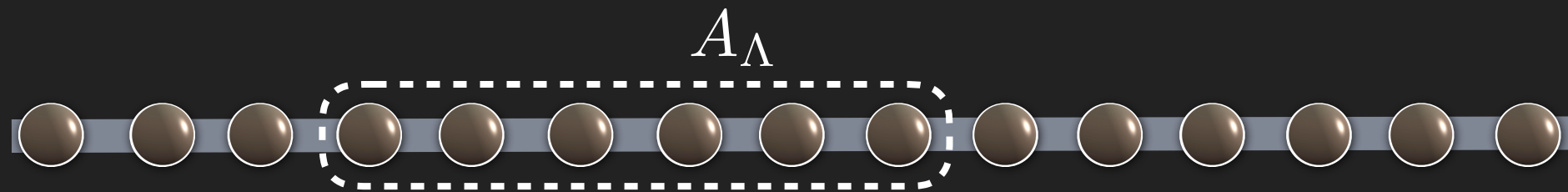
► **Entanglement ergodicity:** $S_2(\rho_{A_\Lambda}(e)) \geq g(e)N$



► **Equilibration:** For any state ρ in the same phase as a product, with energy density e , and a Hamiltonian with non-degenerate gaps in spatial dimension ν , there exists constants C and $k(e) > 0$ such that

$$\text{Var}(O, H_\Lambda, \rho) \leq \|O\|^2 C e^{-k(e)N/(\nu+1)}$$

LESSON



- ▶ **Lesson:** From (very plausible) entanglement ergodicity, general strong equilibration follows



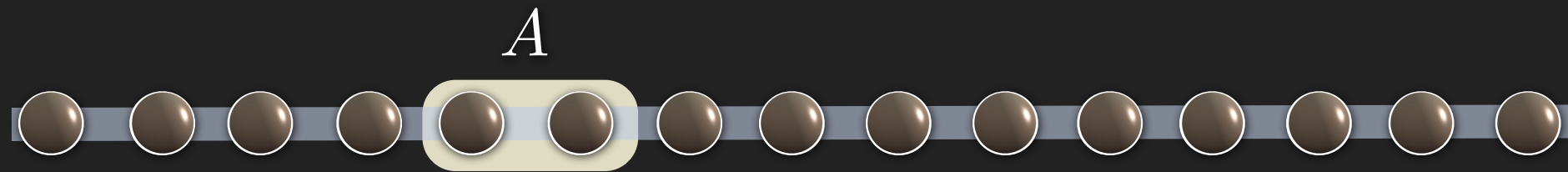
▶ **ETH**

GAUSSIFICATION AND A COLD ATOMIC EXPERIMENT

In preparation

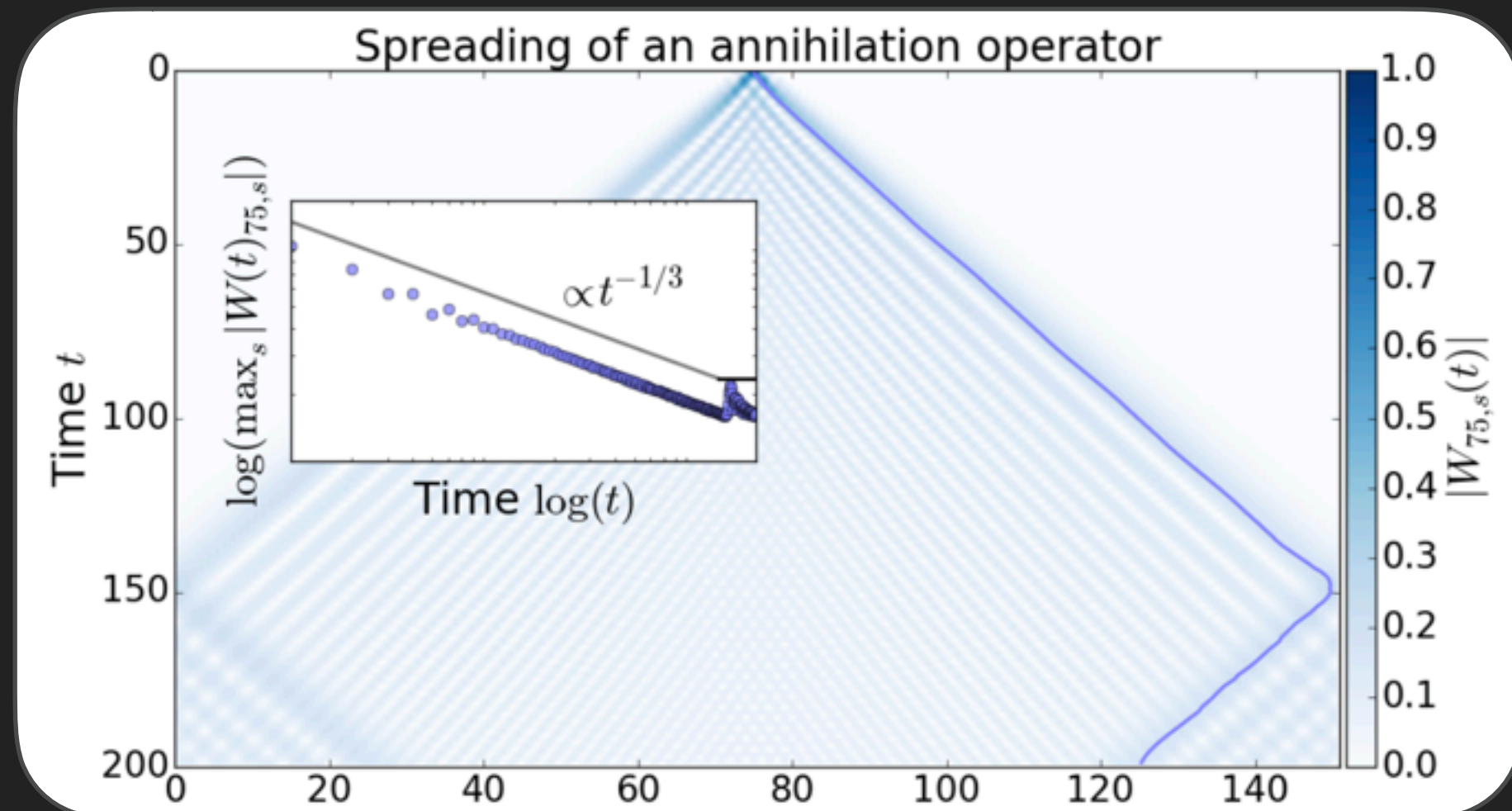


GAUSSIFICATION



↑ Reduced states become Gaussian in time,
even if initial states are highly correlated

- ▶ Quenched non-interacting systems **Gaussify** in time



Gluza, Krumnow, Friesdorf, Gogolin, Eisert, PRL 117 (2016)

Cramer, Dawson, Eisert, Osborne, PRL100, 030602 (2008)

Calabrese, Cardy, Phys Rev Lett 96, 136801 (2006)

A

▶ **Gaussification: If**

- ▶ initial states have **clustering correlations**

$$|\text{tr}(\rho AB) - \text{tr}(\rho A)\text{tr}(\rho B)| \leq C|A| |B| e^{-d(A,B)/\xi}$$

- ▶ the Hamiltonian is quadratic and **free-particle ergodic**

then, for any local observable A , any $\varepsilon > 0$, there exists a relaxation time t_{rel} independent of the system size such that for all $t \in [t_{\text{rel}}, t_{\text{rec}}]$

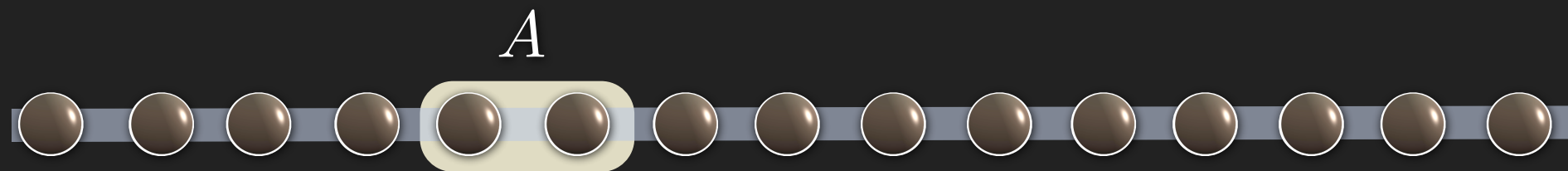
$$|\text{tr}(A(t)\rho) - \text{tr}(A(t)\rho_G)| < \varepsilon$$

where ρ_G is a (possibly time-dependent) **Gaussian state**

Proof techniques: Lieb-Robinson bounds, Bernstein-Spohn blocking, fermionic Lindeberg central limit theorem

- ▶ True for all planar lattices, non-Gaussian initial state, **bosons** and **fermions**

GAUSSIFICATION



GREAT, BUT WHAT DO THE SECOND MOMENTS DO?

A

- ▶ **Free-particle ergodicity:** A system is ergodic if there exists a time t^* such that for all $t \in [t^*, cL]$, the propagator is suppressed as

$$|W_{j,k}(t)| < Ce^{-\alpha t}$$

for some $\alpha > 0$

$$f_j(t) = \sum_{k=1}^n W_{j,k}(t) f_k$$

WHEN IS THIS THE CASE?

A

- **Free-particle ergodicity:** A system is ergodic if there exists a time t^* such that for all $t \in [t^*, cL]$, the propagator is suppressed as

$$|W_{j,k}(t)| < Ce^{-\alpha t}$$

for some $\alpha > 0$



- **Kuzmin theorem:** Suppose (a_n) are real numbers and the gaps $\delta_n = (a_{n+1} - a_n)$ are (i) increasing and (ii) satisfy $\delta_n \in [\lambda, 2\pi - \lambda]$ with $\lambda > 0$, then

$$\left| \sum_{n=0}^{N-1} e^{ia_n} \right| \leq \cot(\lambda/4) \leq \frac{2\pi}{\lambda}$$

- **Always,** for all translationally invariant local models*

Gluza, Eisert, Farrelly, in preparation

* Under very mild assumptions, basically the dispersion relation must not be flat and no points with $E''(p) = E'''(p) = 0$

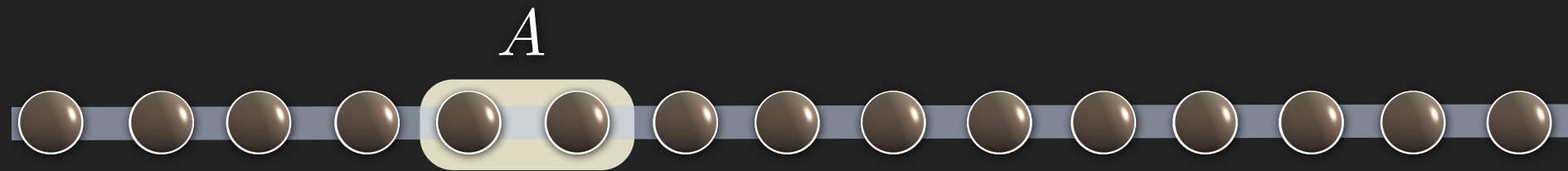


- ▶ **Convergence to GGE:** For generic free fermionic/bosonic TI and Hamiltonians and short range correlated states, one finds convergence to a **Gaussian GGE**,

$$\|\rho_A(t) - \rho_G(t)\|_1 = O(t^{-1})$$

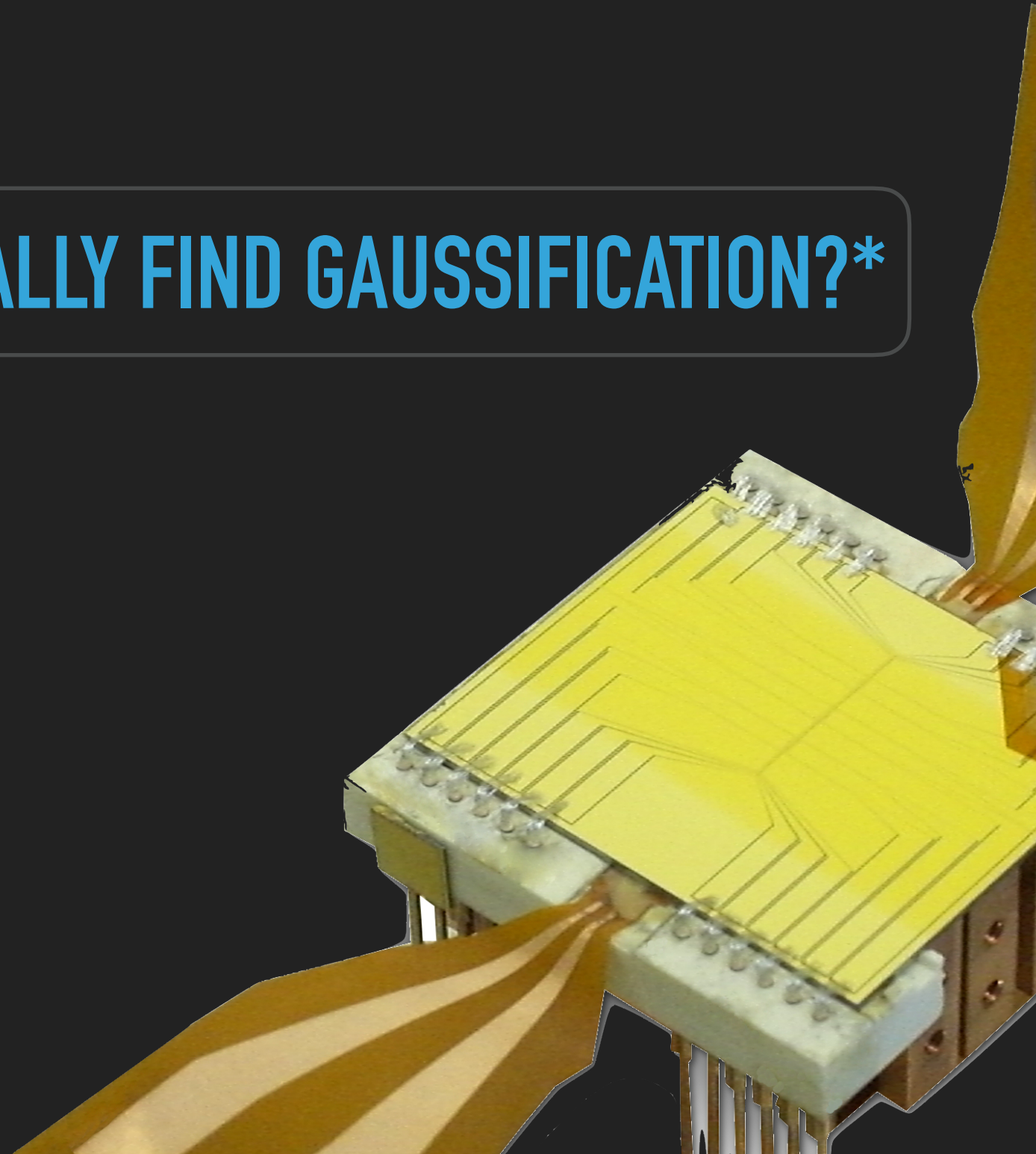
Largely generalizes beautiful work by Calabrese, Essler, Fagotti, PRL 106, 227203 (2011)

- ▶ No an
- ▶ **Lesson:** Get here the full picture of equilibration and GGE convergence, including large independence of initial conditions

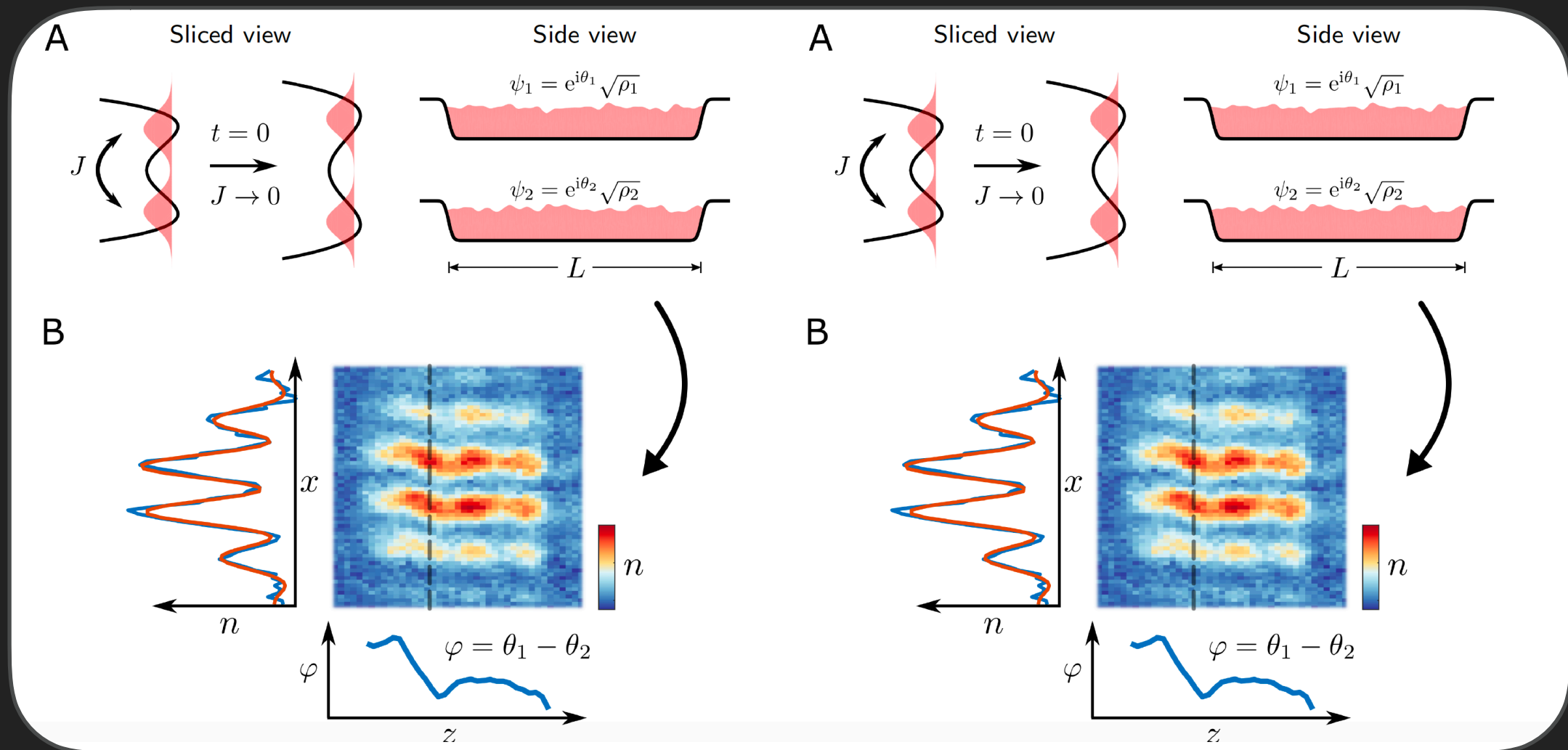


CAN ONE EXPERIMENTALLY FIND GAUSSIFICATION?*

* Joerg Schmiedmayer,
Thomas Schweigler et al



- Yes, in the quantum field states of **cold atoms on atom chips**



- Density and phase quadratures $\psi(z) \sim \sqrt{n_{\text{GP}}(z) + \delta\rho(z)} e^{i\phi(z)}$

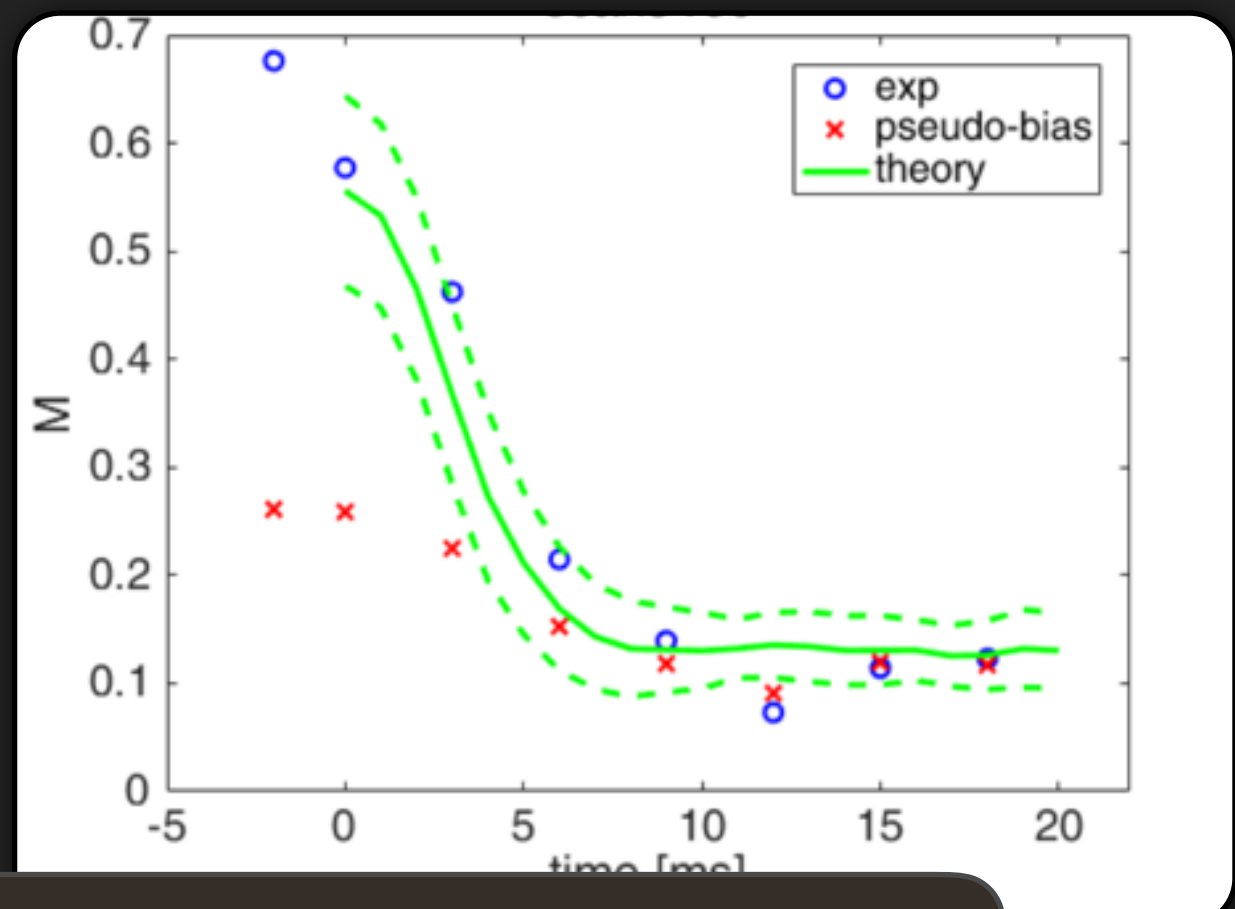
$$[\delta\rho(x), \phi(y)] = i\delta(x - y)$$

► Observe **Gaussification** from higher moments

Connected correlation function

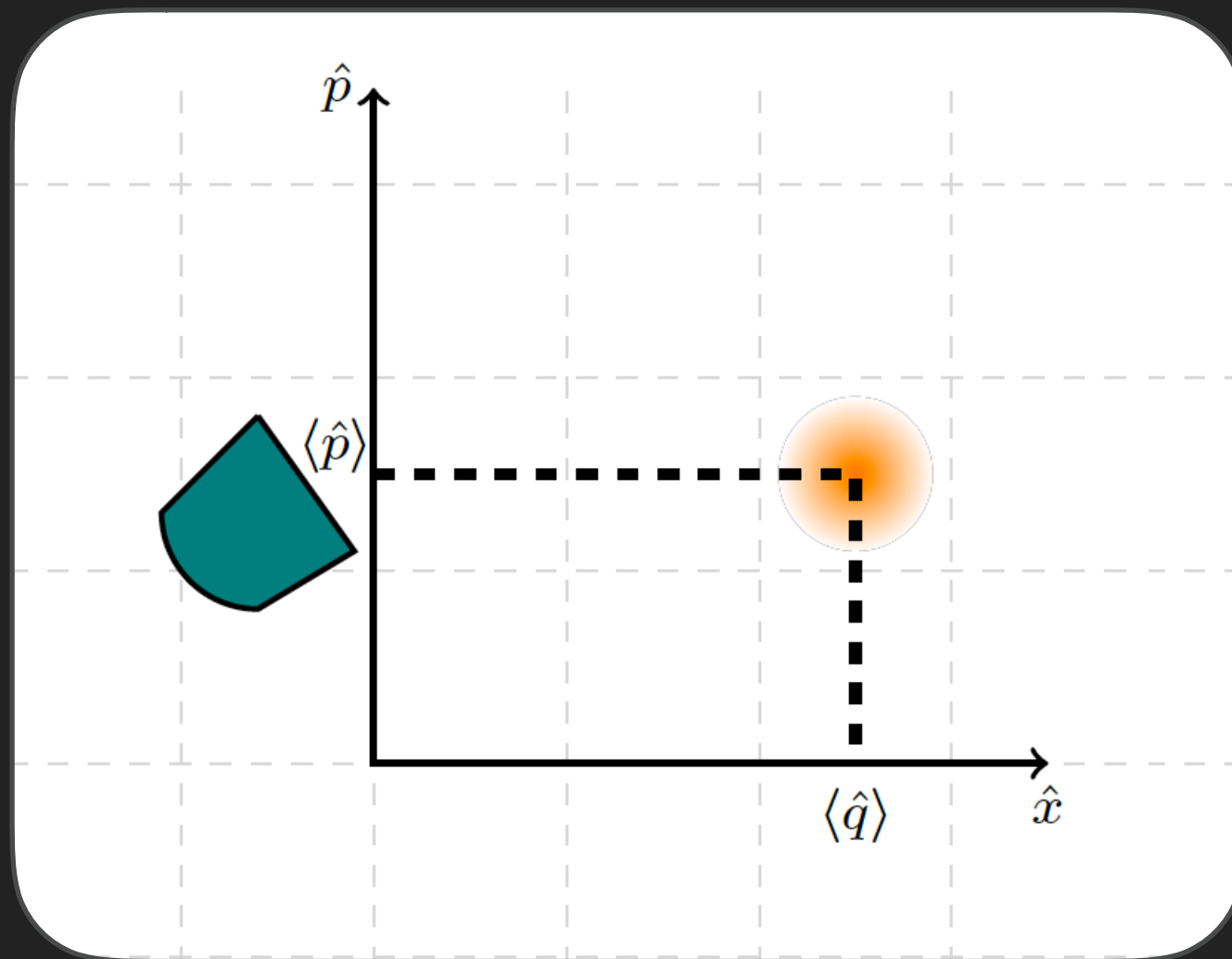
$$M = \frac{\sum_z |W^{(4)}(z)|}{\sum_z |Z^{(4)}(z)|}$$

Full correlation function



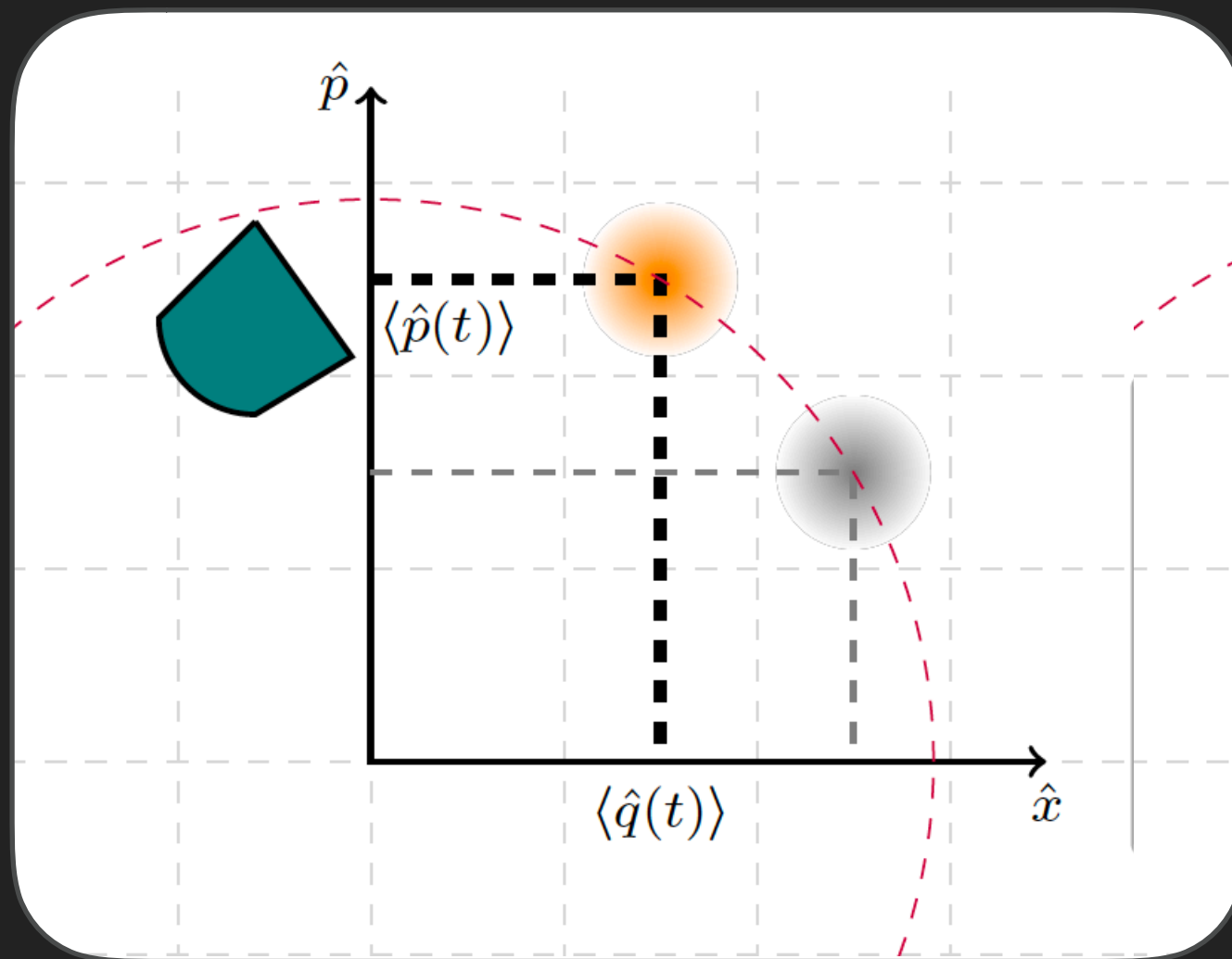
HOW CAN ONE MEASURE THE “MISSING” QUADRATURES?

- Evolution in effective non-interacting (of sine-Gordon) model



$$H = \int_0^L dz \left(\frac{n_{\text{GP}}(z)}{4m} (\partial_z \phi(z))^2 + g \delta \rho(z)^2 \right) = \sum_{k>0} (\omega_k (\phi_k^2 + \delta_k^2) + g \delta \phi_0^2)$$

- Evolution in effective non-interacting (of sine-Gordon) model



$$H = \int_0^L dz \left(\frac{n_{\text{GP}}(z)}{4m} (\partial_z \phi(z))^2 + g \delta \rho(z)^2 \right) = \sum_{k>0} (\omega_k (\phi_k^2 + \delta_k^2) + g \delta \phi_0^2)$$

▶ Make it a **convex** (SDP) **recovery problem**

▶ Measure many
phases in time slices



▶ Evolve under
effective model



▶ Find most likely density
profile in 2-norm, under
Heisenberg constraint

$$\gamma + i\sigma \geq 0$$

RECOVERY OF ALL QUADRATURES

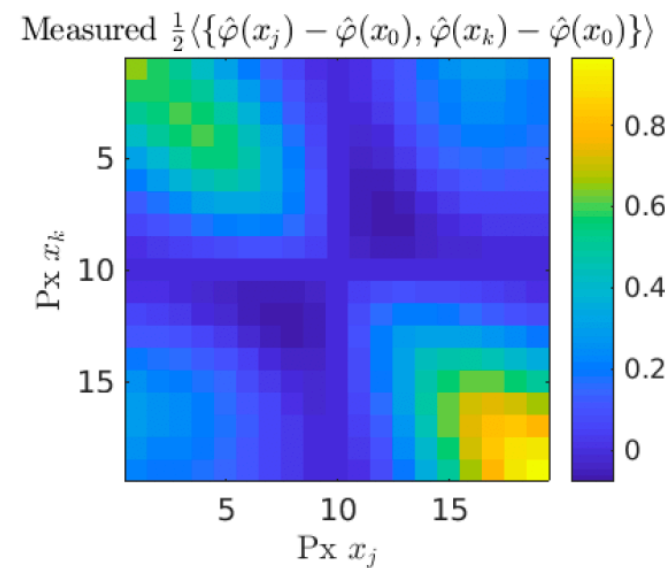
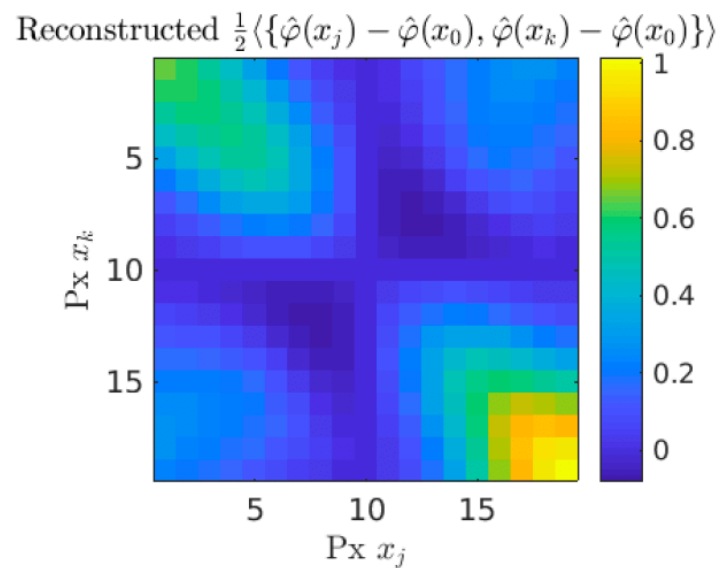
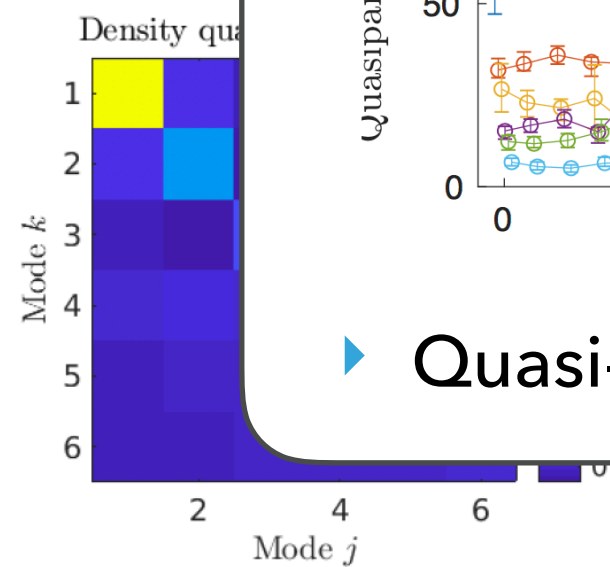
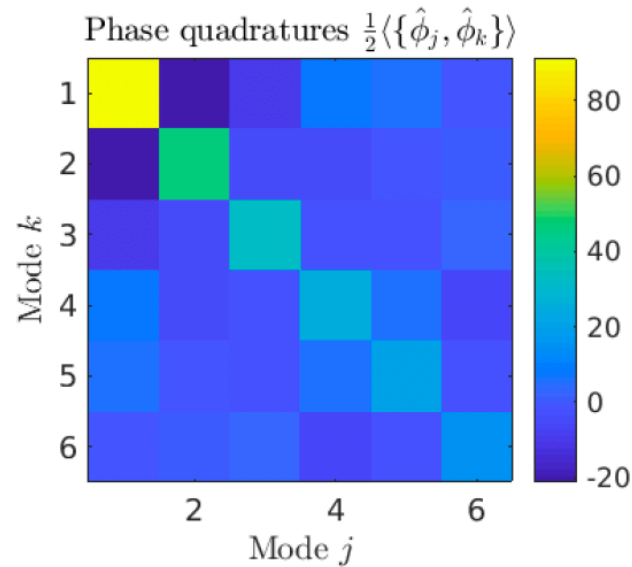
► Make

► Measure m
phases in t

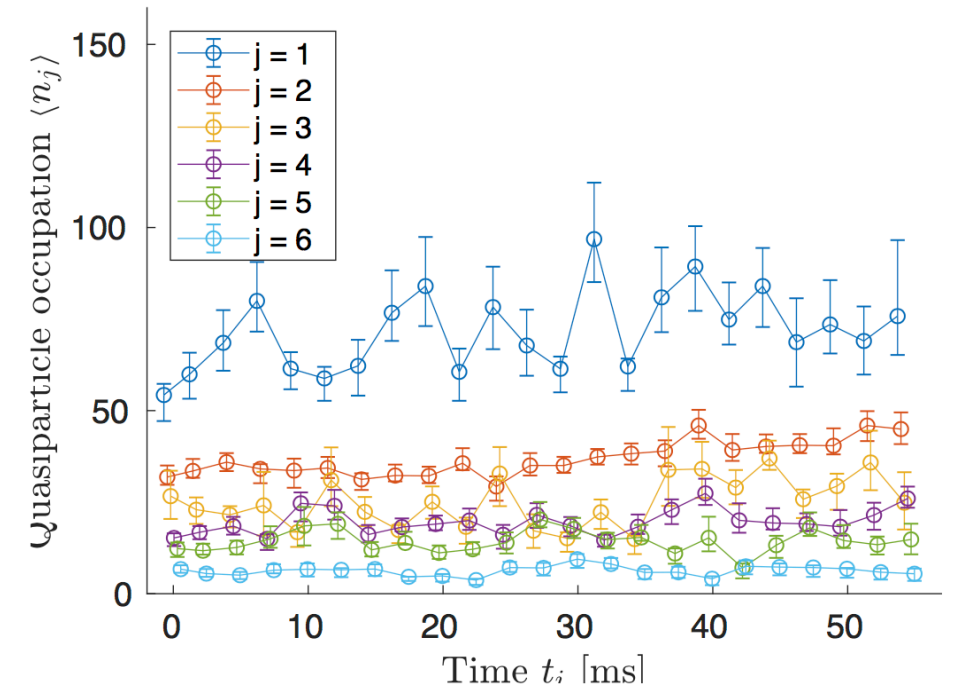
► Evolve und
effective m

► Find most
profile in 2
Heisenberg constraint

$$\gamma + i\sigma \geq 0$$

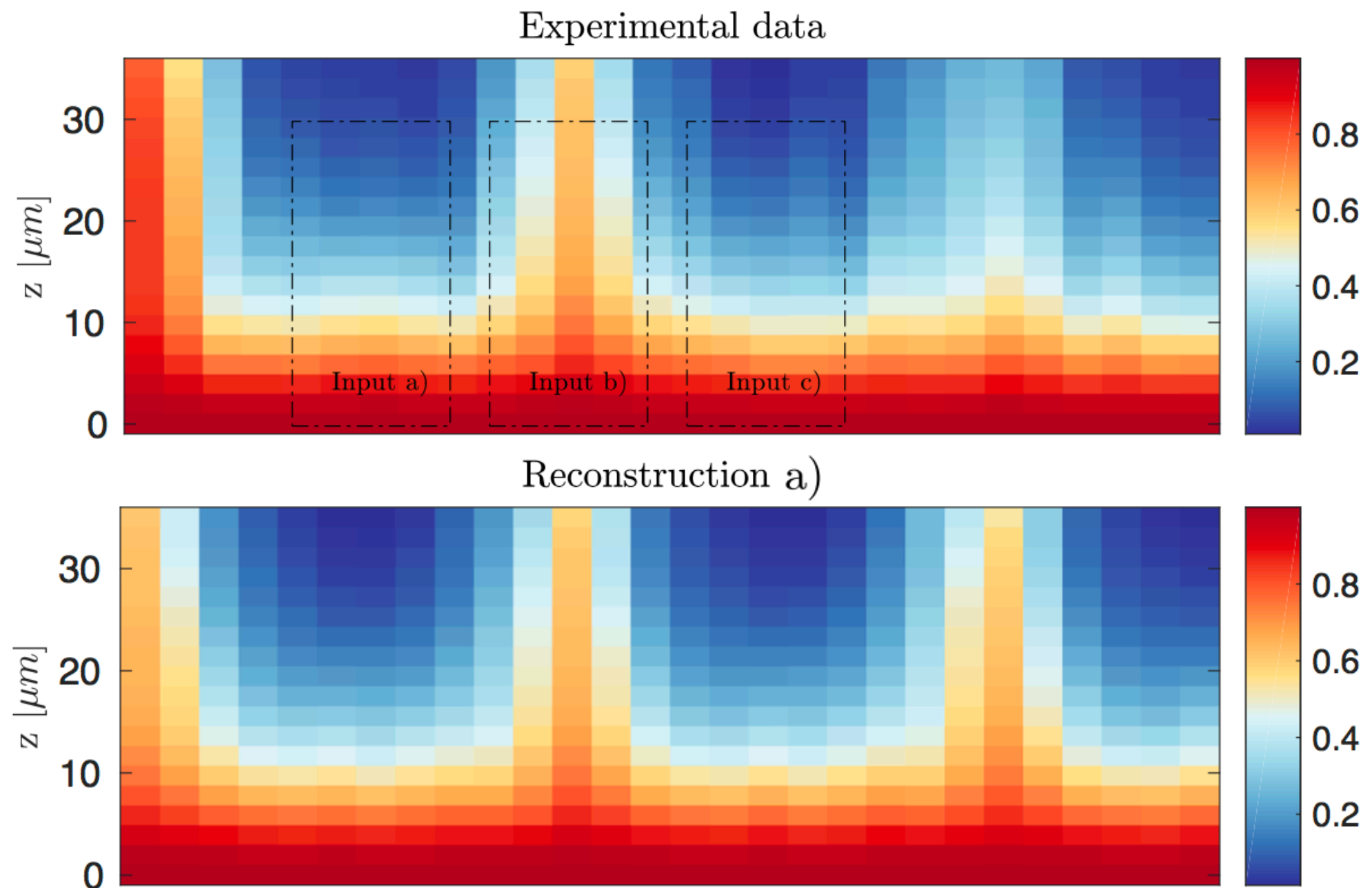


► Quasi-particle occupation



RECOVERY OF ALL QUADRATURES

- Works very well: E.g., **recurrences** in quenched quantum systems



- ▶ **Lesson:** Gaussification is observed; but equally interesting is a new **window** into **cold atomic quantum simulators**
- ▶ Quadratures in 1D bosons can be **measured**
- ▶ Consistent with interacting thermal state preparation
- ▶ Coming now: **Entanglement following quenches**

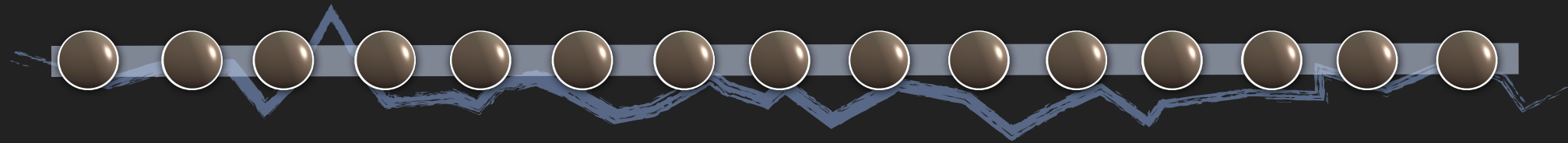
MBL AND THE ABSENCE OF THERMALIZATION

arXiv:1707.05181

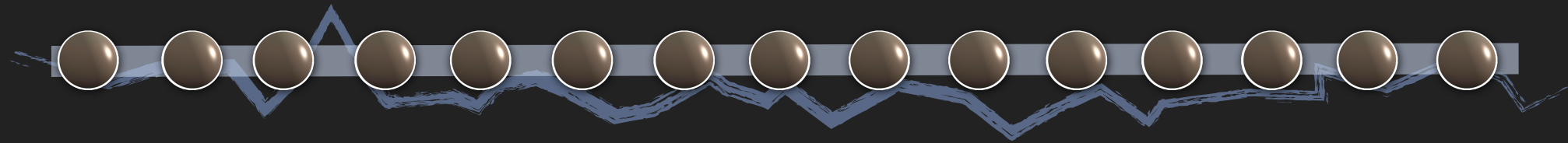
In preparation



MANY-BODY LOCALIZATION: INTERPLAY OF DISORDER AND INTERACTION



- ▶ Some systems stubbornly **refuse** to thermalize
- ▶ **Many-body localization:** Interplay of disorder and interactions



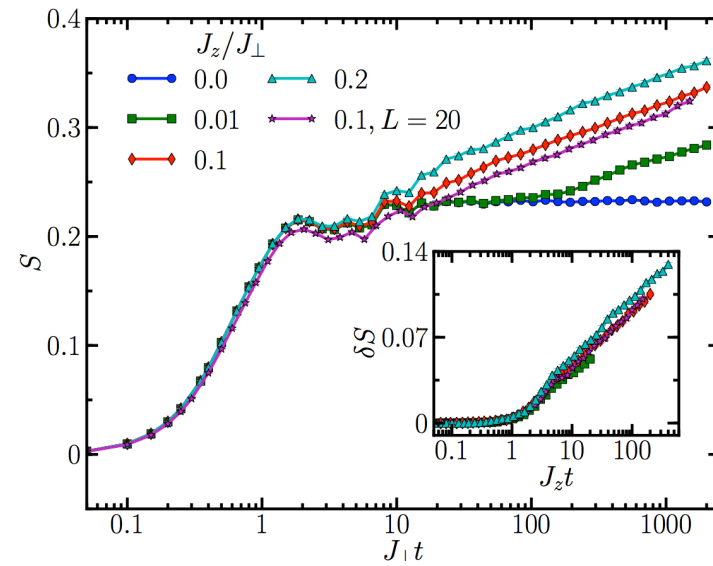
- ▶ *Anderson model*: Particle hopping on a line under random potential

$$H = \sum_j (|j\rangle\langle j+1| + |j+1\rangle\langle j| + f_j |j\rangle\langle j|)$$

- ▶ Static localization: Most eigenstates have clustering correlations
- ▶ Dynamic localization: $\mathbb{E}(\sup_t |\langle n | e^{-itH} | m \rangle|) \leq c_1 e^{-c_2 \text{dist}(n,m)}$

- ▶ **Many-body localization**: Rich phenomenology, still a crime story

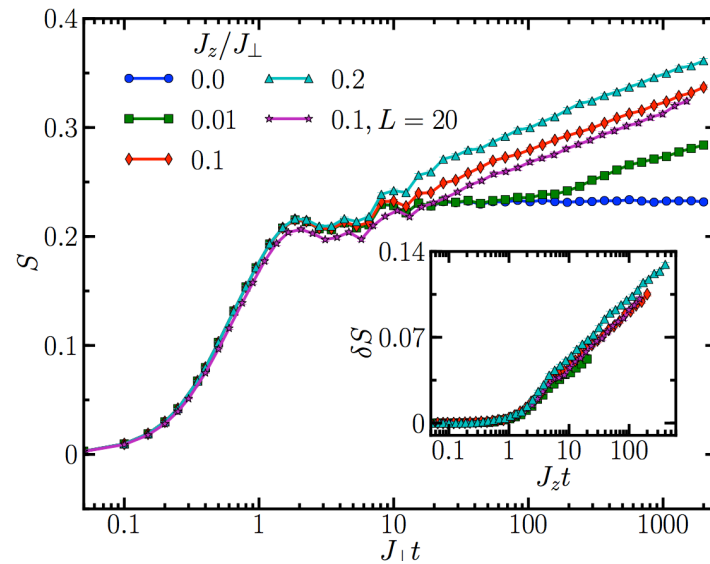
► Log-growth of entanglement



Znidaric, Prosen, Prelovsek, PRB 77, 064426 (2008)
Badarson, Pollmann, Moore, PRL 109, 017202 (2012)

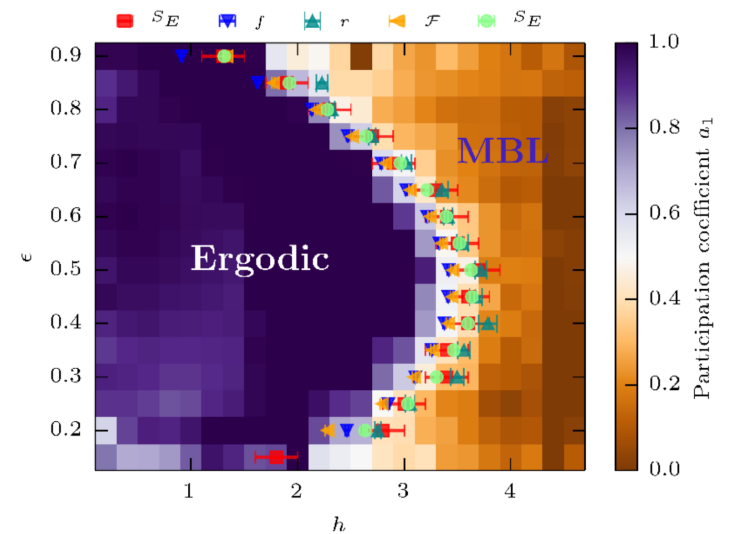
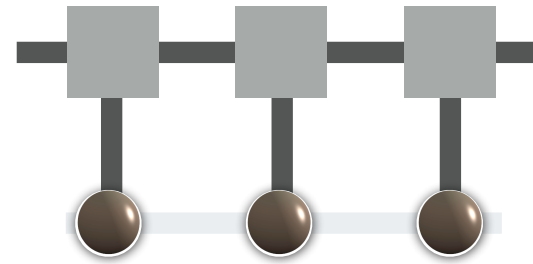
OF DISORDER AND INTERACTION

► Log-growth of entanglement



Znidaric, Prosen, Prelovsek, PRB 77, 064426 (2008)
 Badarson, Pollmann, Moore, PRL 109, 017202 (2012)

► Matrix product, area-law eigenstates



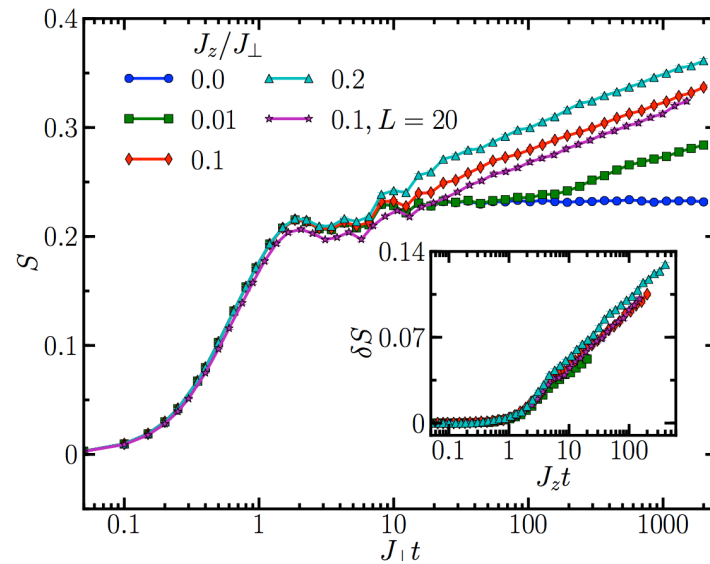
Bauer, Nayak, J Stat Mech P09005 (2013)
 Luitz, Laflorencie, Alex, arXiv:1411.0660

► Static localization follows from dynamical one

$$\|A(t) - e^{itH_A^l} A e^{-itH_A^l}\| \leq c_{\text{loc}} e^{-\mu(l+c_2 \log(t))}$$

Friesdorf, Werner, Scholz, Brown, Eisert, Phys Rev Lett 114, 170505 (2015)
 In preparation (2018)

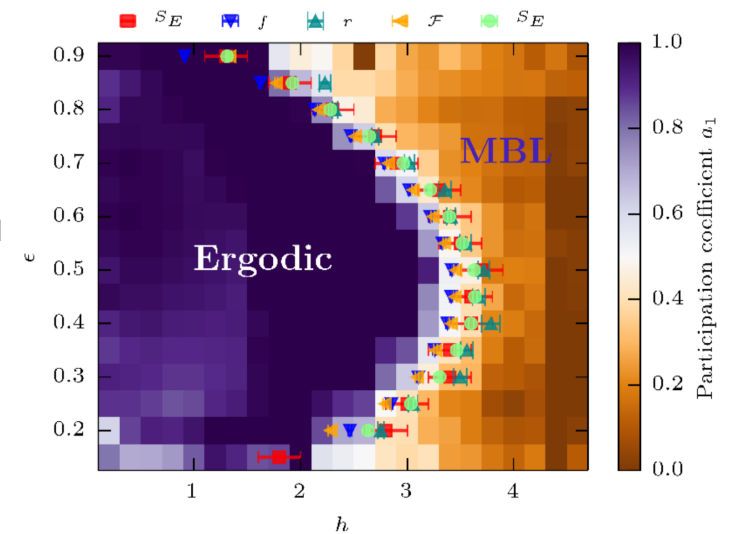
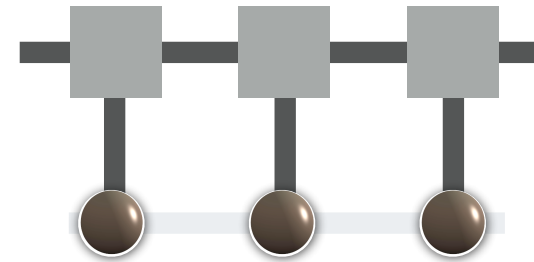
► Log-growth of entanglement



Znidaric, Prosen, Prelovsek, PRB 77, 064426 (2008)
 Badarson, Pollmann, Moore, PRL 109, 017202 (2012)

Kim, Chandran, Abanin, arXiv:1412.3073
 Eisert, Osborne, Phys Rev Lett 97, 150404 (2006)

► Matrix product, area-law eigenstates



Bauer, Nayak, J Stat Mech P09005 (2013)
 Luitz, Laflorencie, Alex, arXiv:1411.0660

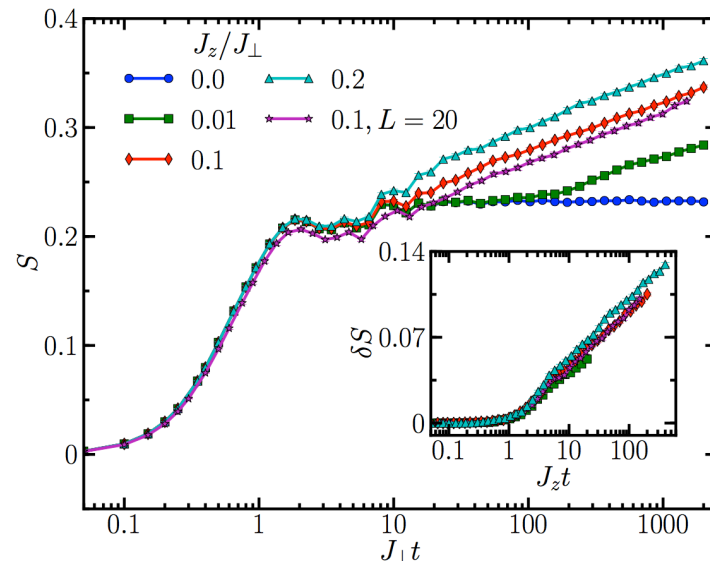
Friesdorf, Werner, Goihl, Eisert, Brown, NJP 17, 113054 (2015)
 Chandran, Carresquilla, Kim, Abanin, Vidal, PRB 92, 024201 (2015)

► “I-bit Hamiltonian” in terms of quasi-local com

$$H_{\text{eff}}^{(N_{\text{eff}})} = \sum_i \omega_i^{(1)} \tau_i^z + \sum_{i,j} \omega_{i,j}^{(2)} \tau_i^z \tau_j^z + r$$

Huse, Nandkishore, Oganesyan, Phys Rev B 90, 174202 (2014)

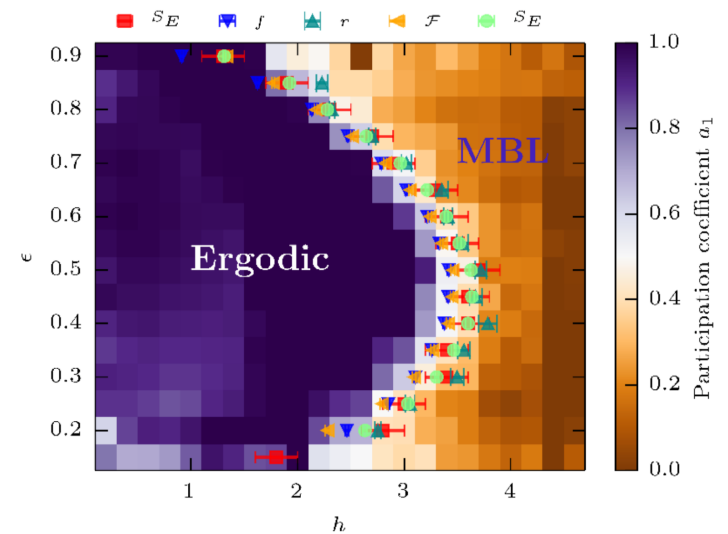
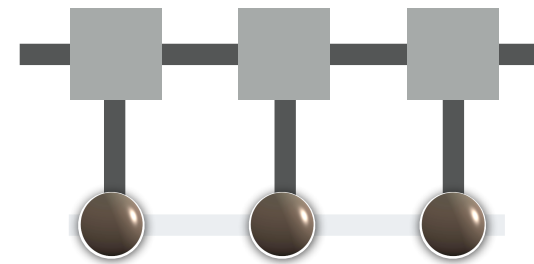
▶ Log-growth of entanglement



Znidaric, Prosen, Prelovsek, PRB 77, 064426 (2008)
 Badarson, Pollmann, Moore, PRL 109, 017202 (2012)

Kim, Chandran, Abanin, arXiv:1412.3073
 Eisert, Osborne, Phys Rev Lett 97, 150404 (2006)

▶ Matrix product, area-law eigenstates



Bauer, Nayak, J Stat Mech P09005 (2013)
 Luitz, Laflorencie, Alex, arXiv:1411.0660

Friesdorf, Werner, Goihl, Eisert, Brown, NJP 17, 113054 (2015)
 Chandran, Carresquilla, Kim, Abanin, Vidal, PRB 92, 024201 (2015)

▶ “I-bit Hamiltonian” in terms of quasi-local com

$$H_{\text{eff}}^{(N_{\text{eff}})} = \sum_i \omega_i^{(1)} \tau_i^z + \sum_{i,j} \omega_{i,j}^{(2)} \tau_i^z \tau_j^z + r$$

Huse, Nandkishore, Oganesyan, Phys Rev B 90, 174202 (2014)

Friesdorf, Werner, Goihl, Eisert, Brown, NJP 17, 113054 (2015)

Brown, Goihl, Werner, Eisert, im preparation

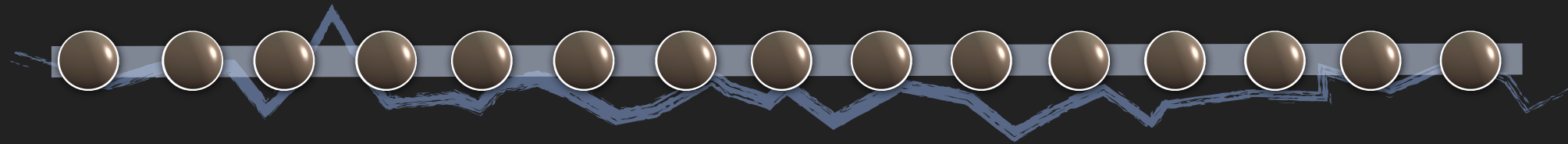
▶ Slow information propagation

Kim, Banuls, Cirac, Hastings, Huse, PRE 92, 012128 (2015)

▶ Slow equilibration

Brown, Goihl, Werner, Eisert, im preparation

MANY-BODY LOCALIZATION: INTERPLAY OF DISORDER AND INTERACTION



► **"l-bit Hamiltonian"** in terms of quasi-local com

$$H_{\text{eff}}^{(N_{\text{eff}})} = \sum_i \omega_i^{(1)} \tau_i^z + \sum_{i,j} \omega_{i,j}^{(2)} \tau_i^z \tau_j^z + r$$

BUT HOW TO FIND L-BIT HAMILTONIAN?

Vosk, Altman, PRL 110, 067204 (2013)

Rademaker, Ortuno, PRL 116, 010404 (2016)

Pekker, Clark, Oganesyan, Refael, 1607.07884

NUMERICALLY FINDING L-BIT HAMILTONIANS



- ▶ **Want:** Representation of Pauli algebra (mutually commuting)
- ▶ Commuting with Hamiltonian

NUMERICALLY FINDING L-BIT HAMILTONIANS



- ▶ Start from energy eigenbasis $\{|e\rangle\}$
- ▶ Find rationale to permute $|k\rangle = \{|\pi(e)\rangle\}$, $\pi \in S_D, D = 2^L!$

NUMERICALLY FINDING L-BIT HAMILTONIANS



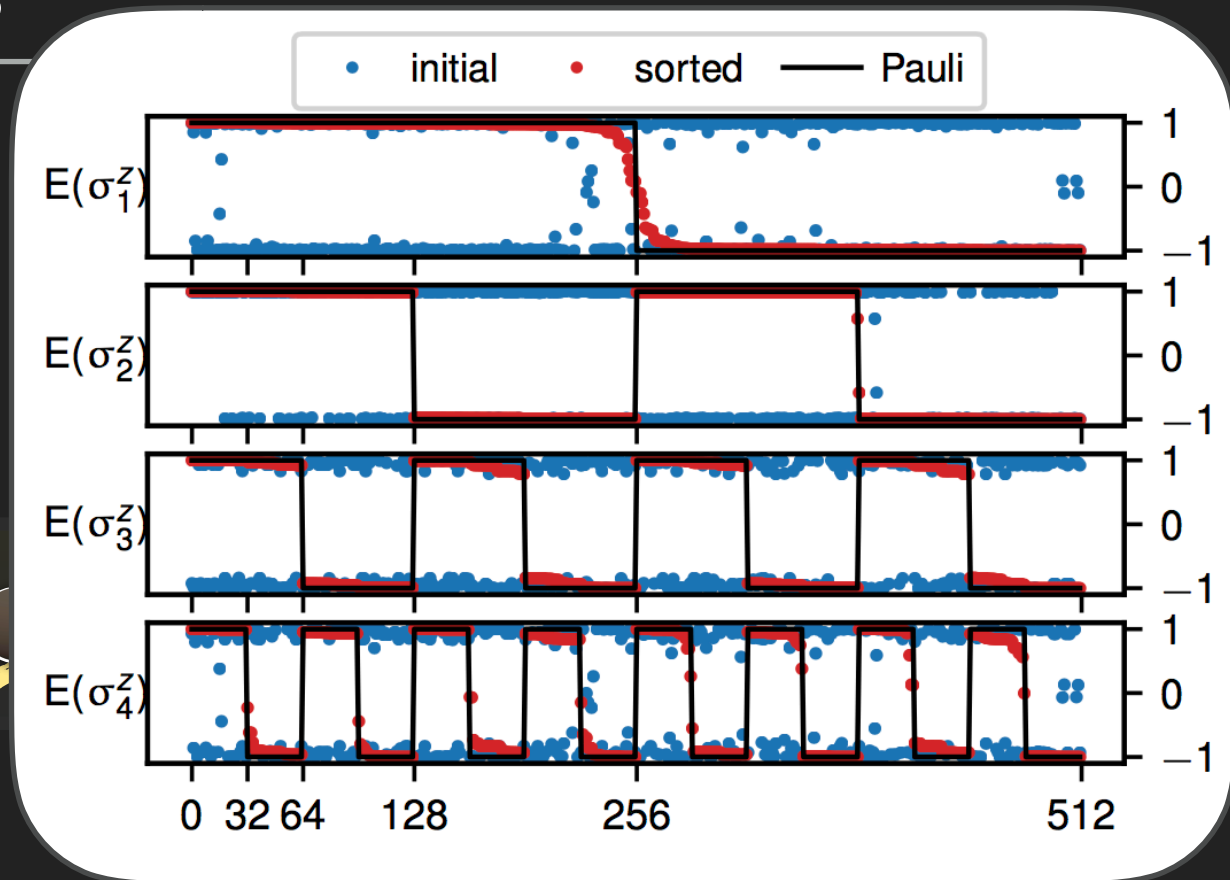
- ▶ Start from $\mathcal{Z}^{(1)} = \sigma_z \otimes 1_{2^{L-1}}$, taking form $\mathcal{Z}^{(1)} = 1_{2^{L-1}} \oplus (-1_{2^{L-1}})$
- ▶ Order $|\pi_1(e)\rangle$ so that diag of infinite time average

$$\bar{\sigma}_z^{(1)}(t) = \sum_e \langle e | \sigma_z^{(1)} | e \rangle | e \rangle \langle e |$$

is ordered decreasingly

- ▶ Next $\mathcal{Z}^{(2)} = 1_{2^{L-2}} \oplus (-1_{2^{L-1}}) \oplus 1_{2^{L-2}} \oplus (-1_{2^{L-1}})$, order $|\pi_2 \circ \pi_1(e)\rangle$ so that $\bar{\sigma}_z^{(2)}$ is ordered decreasingly
- ▶ Etc

NUMERICALLY FINDING L-BIT HAMILTONIANS

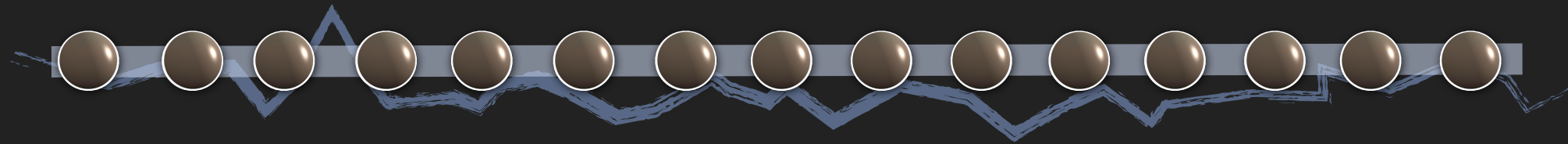


- ▶ Gives surprisingly good energies
- ▶ Orthogonalize Hamiltonian to l-bit form
- ▶ Decay in $\|\cdot\|_2$ -norm, can see phase transition exploring support
- ▶ Iterated can be made tensor network (at expense of small errors)

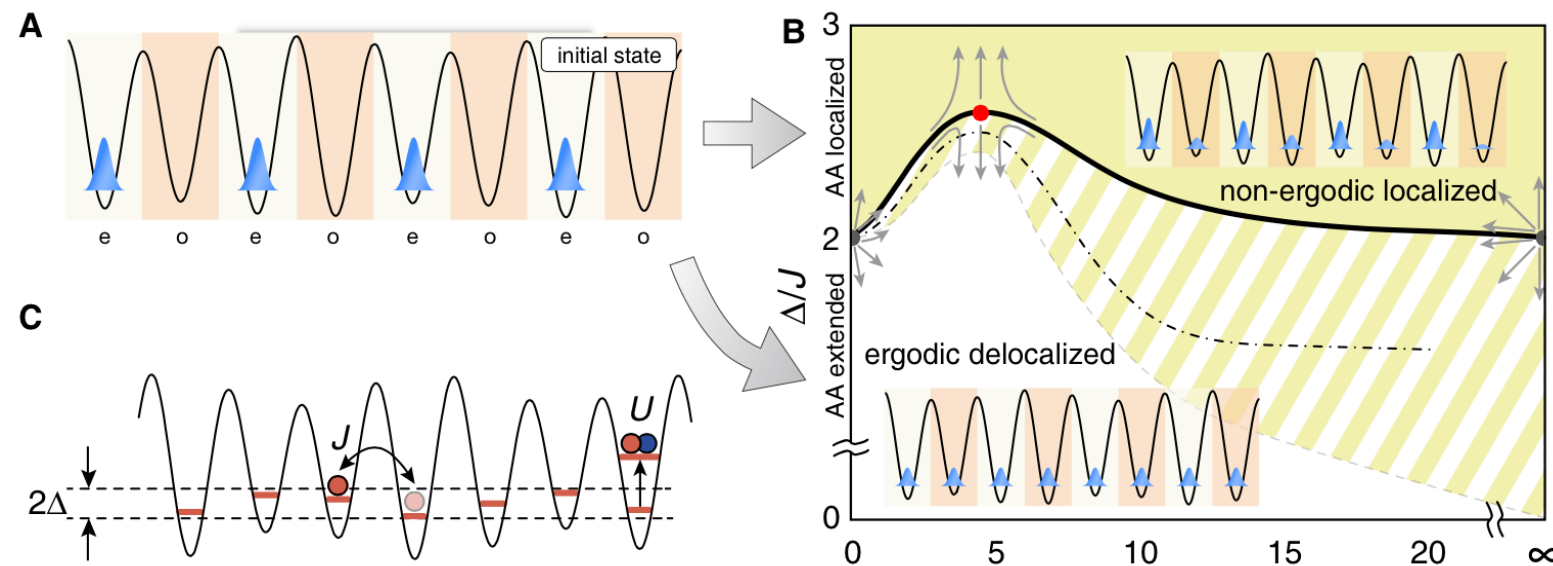
NUMERICALLY FINDING L-BIT HAMILTONIANS



- ▶ **Lesson:** Can obtain l -bit Hamiltonians to good precision with simple method

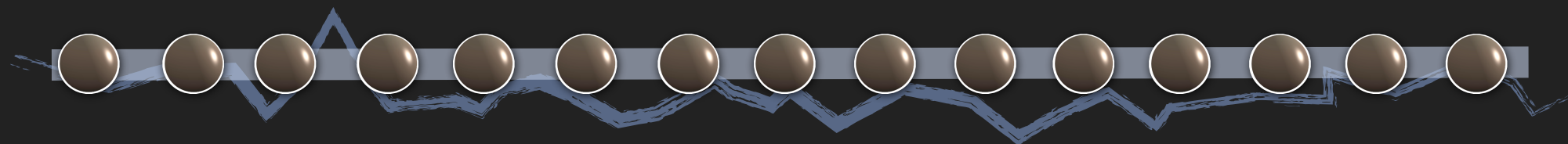


► Cold atomic quantum simulations of many-body localization



Schreiber, Hodgman, Bordia, Lueschen, Fischer, Vask, Altman, Schneider, Bloch, Science 349, 842 (2015)
Choi, Hild, Zeiher, Schreiber, Bloch, Science 352, 1557 (2015)

HOW TO UNAMBIGUOUSLY MEASURE MBL?

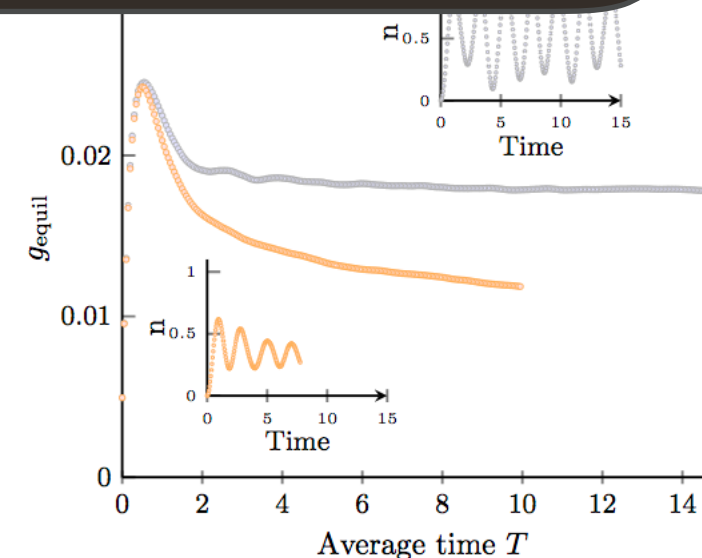
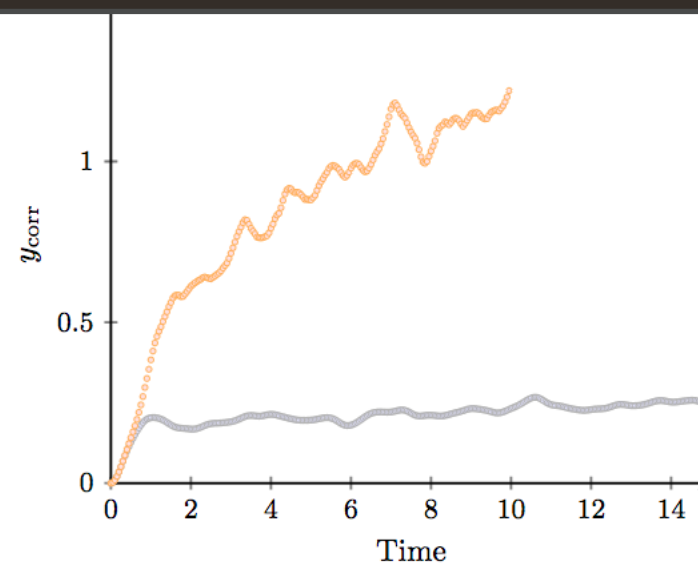
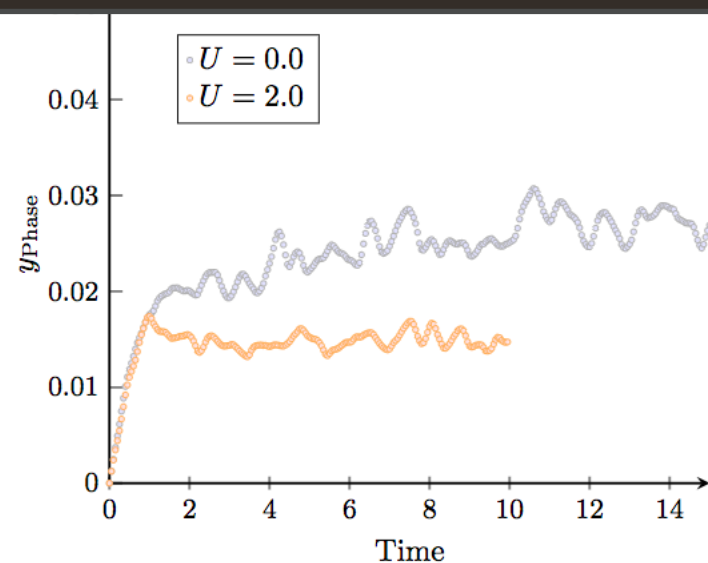


► In-situ and parity-projected density-density correlations

$$f_{\text{Corr}}(k, t) = |\langle n_{L/2} n_{L/2+k} \rangle - \langle n_{L/2} \rangle \langle n_{L/2+k} \rangle|$$

$$y_{\text{Corr}}(k, t) = \sum_{k=1}^T f_{\text{Corr}}(k, t) k^2$$

► **Lesson:** Building on I-bit intuition, can devise **feasible witnesses** of MBL discriminating from Anderson localization



TOWARDS QUANTUM ADVANTAGES

Phys Rev X 8, 021010 (2018)

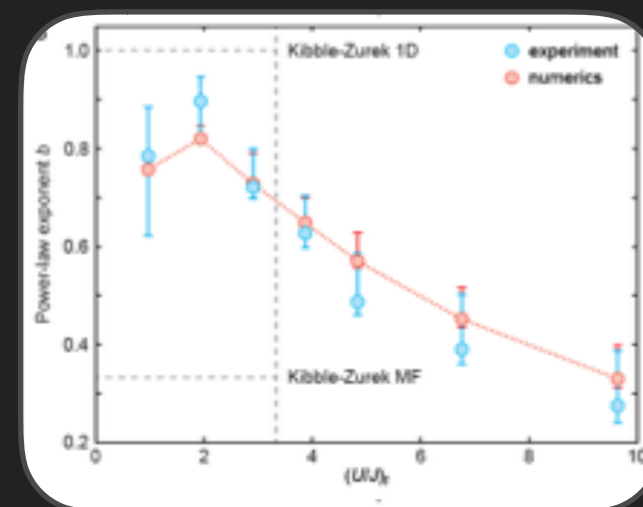
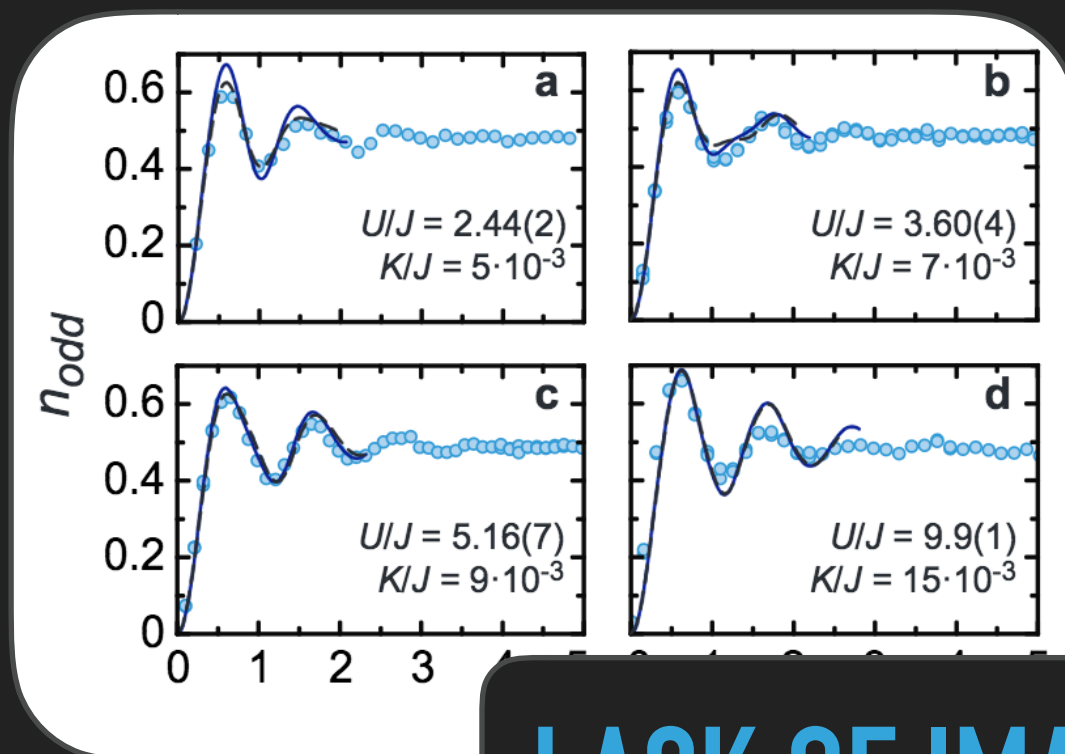
Quantum 2, 65 (2018)



QUANTUM SIMULATORS SHOWING A QUANTUM ADVANTAGE



- ▶ Quest for **quantum advantage** of quantum devices
- ▶ Quantum simulators already outperform state-of-the-art algorithms



Braun, Friesdorf, Hodgman, Schreiber, Ronzheimer, Riera, del Rey, Bloch, Eisert, *Science* 348, 1025 (2015)
Schreiber, Friesdorf, Hodgman, Schreiber, Ronzheimer, Riera, del Rey, Bloch, Eisert, *Science* 348, 1025 (2015)
s, Rubio-Abadal, Yefzah, *Science* 352, 1547 (2016)

Trotzky, Chen, Flesch, Moench, Bloch, *Nature Physics* 8, 325 (2012)

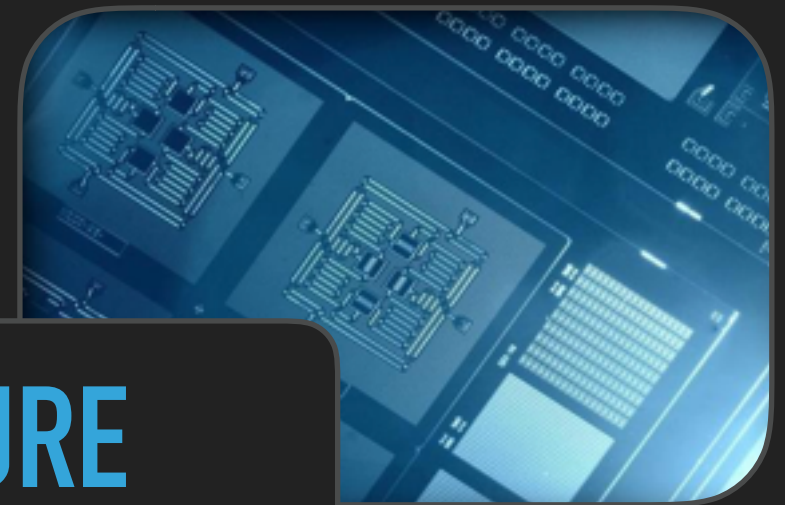
LACK OF IMAGINATION?



- ▶ IBM/Google: **Intermediate problems** for superconducting qubits
- ▶ Boson sampling: Outperforms classical computers in terms of computational complexity

Aaronson, Arkhipov, Th Comp 9, 143 (2013)

Boixo, Isakov, Smelzanski, Babbush, Ding, Jiang, Bremner, Martinis, Neven, arXiv:1608.00263 (2016)



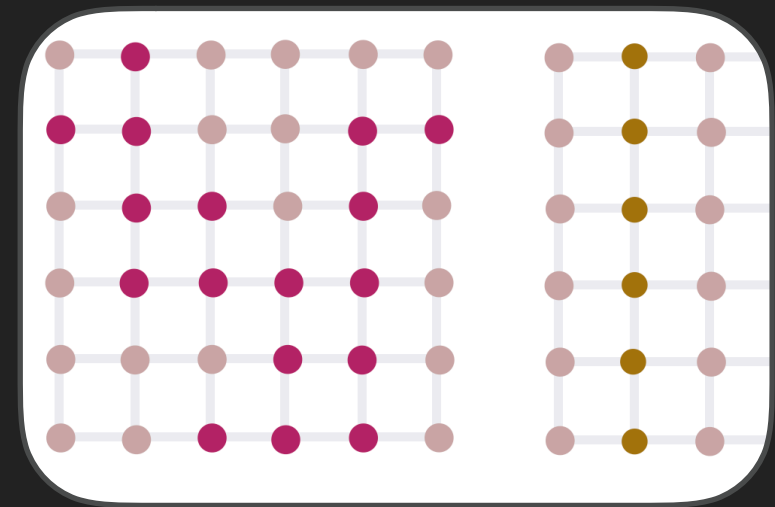
**HOW CAN ONE EVER BE SURE
THAT THEY DO THE RIGHT THING?**

- ▶ **But:** No efficient discrimination from classical devices



- ▶ Devise **cold atomic simulators** showing a **quantum advantage**

- ▶ Prepare **product** state
- ▶ Evolve for unit time under **local** Ham
- ▶ In-situ **measure**



- ▶ Gives *computationally hard* intermediate problem: Sampling is computationally hard up to an additive error in the total variation distance for a classical computer, but...

QUANTUM SIMULATORS SHOWING A QUANTUM ADVANTAGE



- ▶ Devise **cold atomic simulators** showing a **quantum advantage**
- ▶ ... it can be efficiently **certified**
(with local measurements)



SUMMARY

