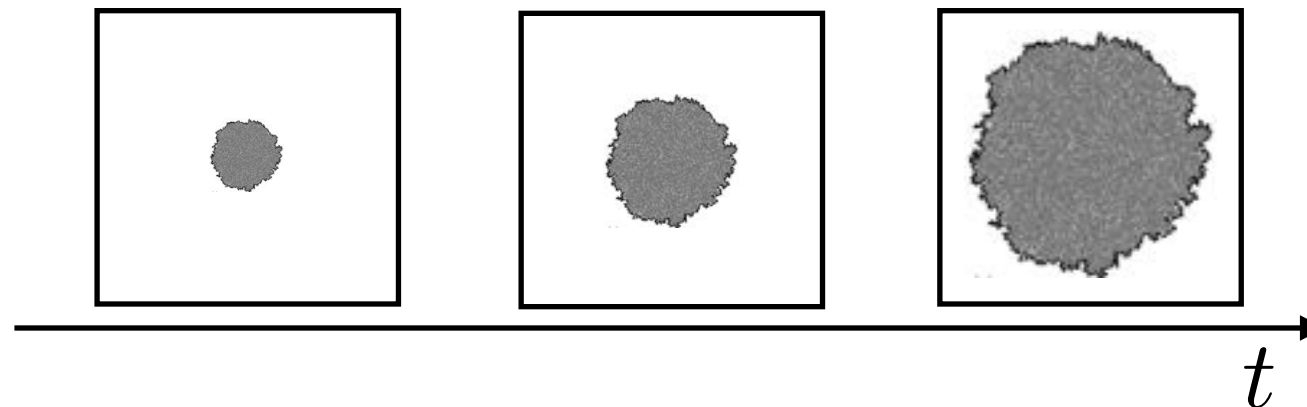
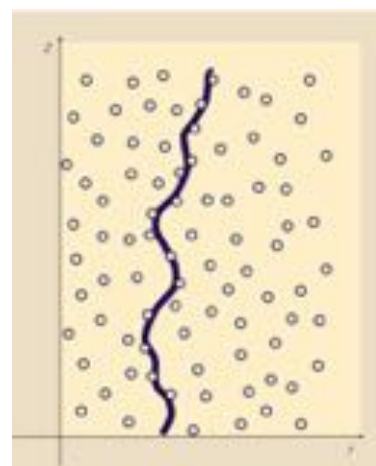


KPZ universality and memory

KPZ universality class



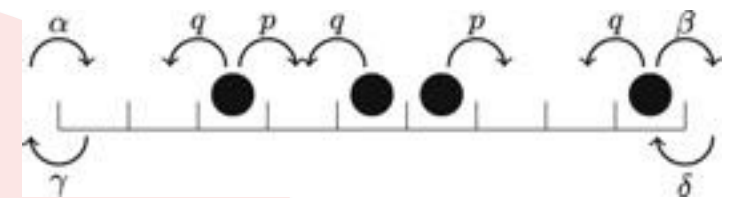
Kardar-Parisi-Zhang (KPZ)



Directed
Polymer

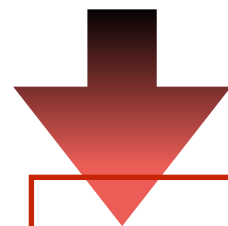
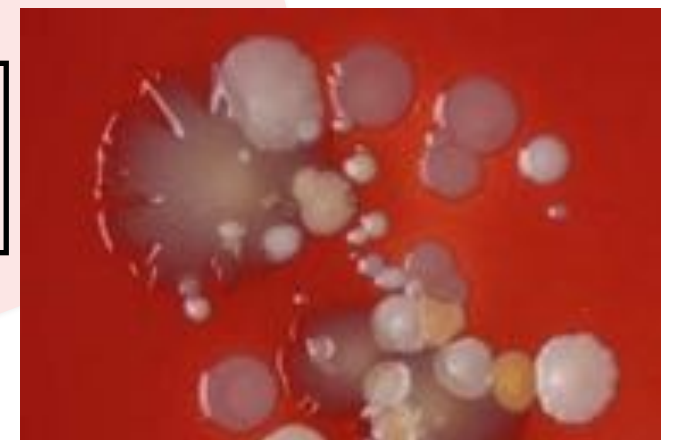
Quantum
systems

Exclusion
Processes



Matrices
aléatoires

Colonies
of bacteria



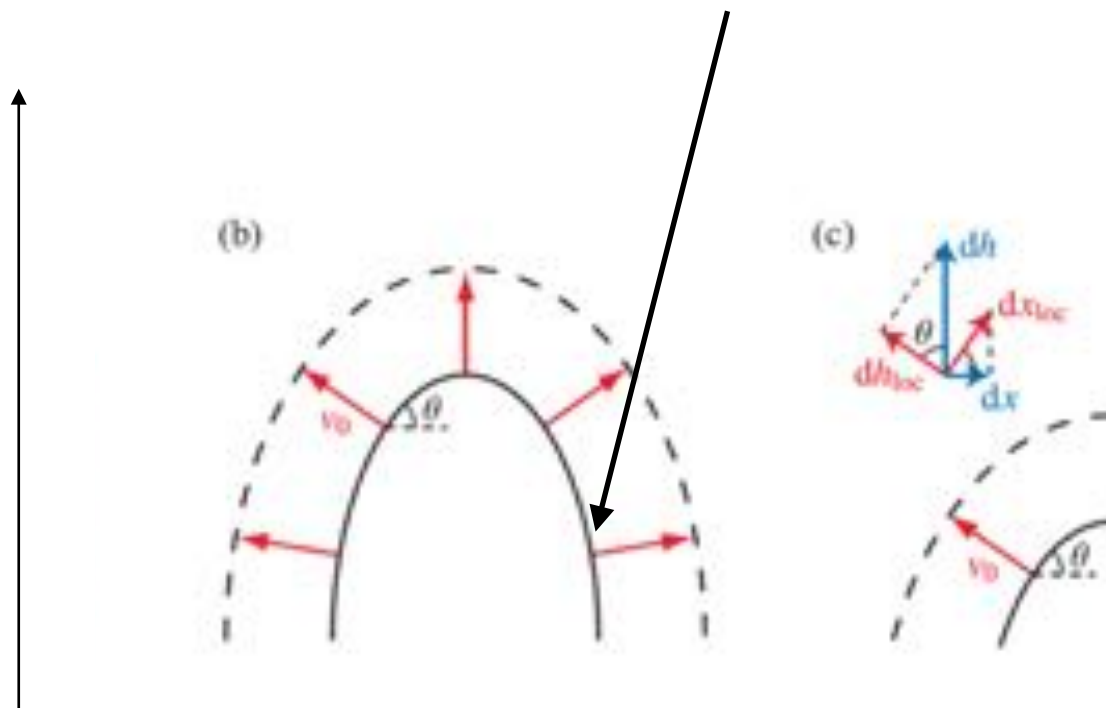
$$\partial_t h(x, t) = \nu \partial_x^2 h(x, t) + \frac{\lambda_0}{2} (\partial_x h(x, t))^2 + \sqrt{D} \xi(x, t)$$

Solved in 2010

KPZ universality class

$$\partial_t h(x, t) = \nu \partial_x^2 h(x, t) + \frac{\lambda_0}{2} (\partial_x h(x, t))^2 + \sqrt{D} \xi(x, t)$$

$$\partial_t h_{\text{local}}(x_{\text{local}}, t) = \nu \partial_{x_{\text{local}}}^2 h_{\text{local}}(x_{\text{local}}, t) + \sqrt{D} \eta(x_{\text{local}}, t)$$



$$dh = dh_{\text{local}} \sqrt{1 + (\partial_x h)^2}$$

$$dx = dx_{\text{local}} / \sqrt{1 + (\partial_x h)^2}$$

An appetizer to modern developments on the Kardar-Parisi-Zhang universality class

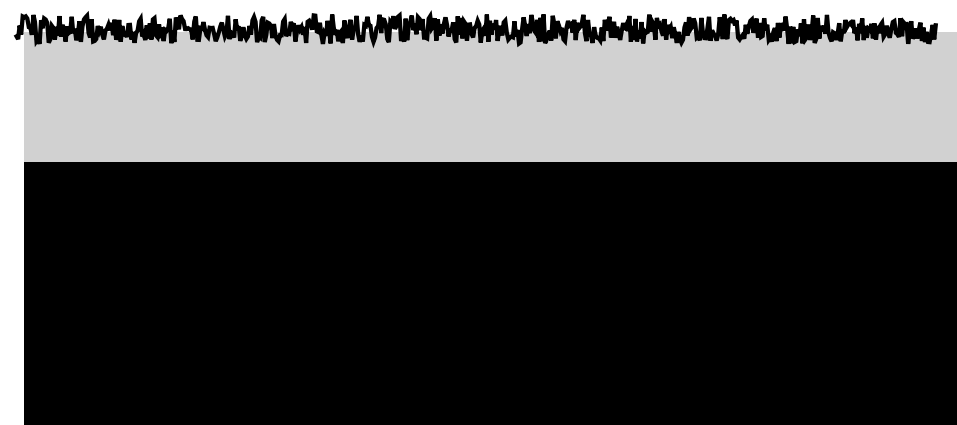
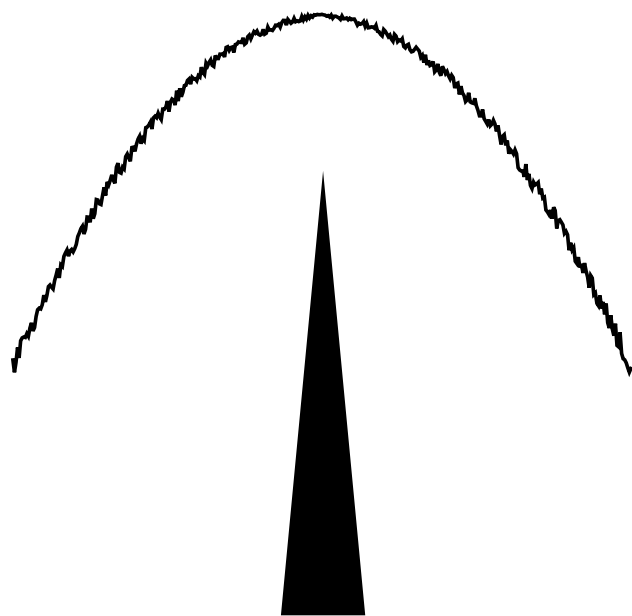
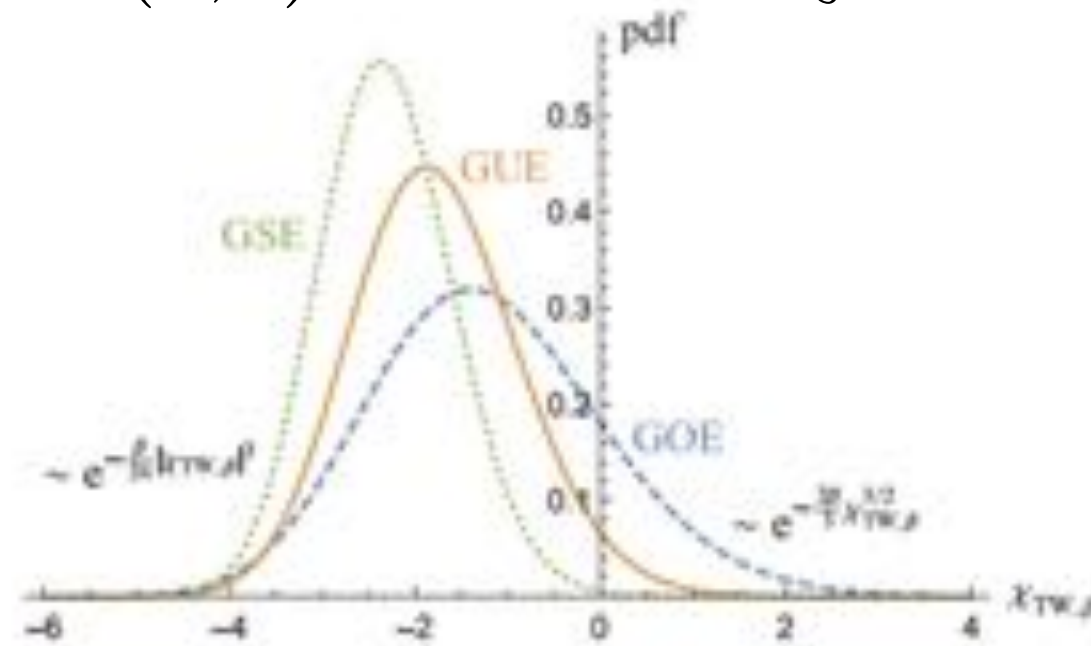
Kazumasa A. Takeuchi

Department of Physics, Tokyo Institute of Technology,
2-12-1 Ookayama, Meguro-ku, Tokyo 152-8551, Japan.

KPZ universality class

$$\partial_t h(x, t) = \nu \partial_x^2 h(x, t) + \frac{\lambda_0}{2} (\partial_x h(x, t))^2 + \sqrt{D} \xi(x, t)$$

$$h(0, t) = vt + t^{1/3} \xi + \dots$$



KPZ universality class - quantum systems

PRL 110, 060602 (2013)

PHYSICAL REVIEW LETTERS

week ending
8 FEBRUARY 2013

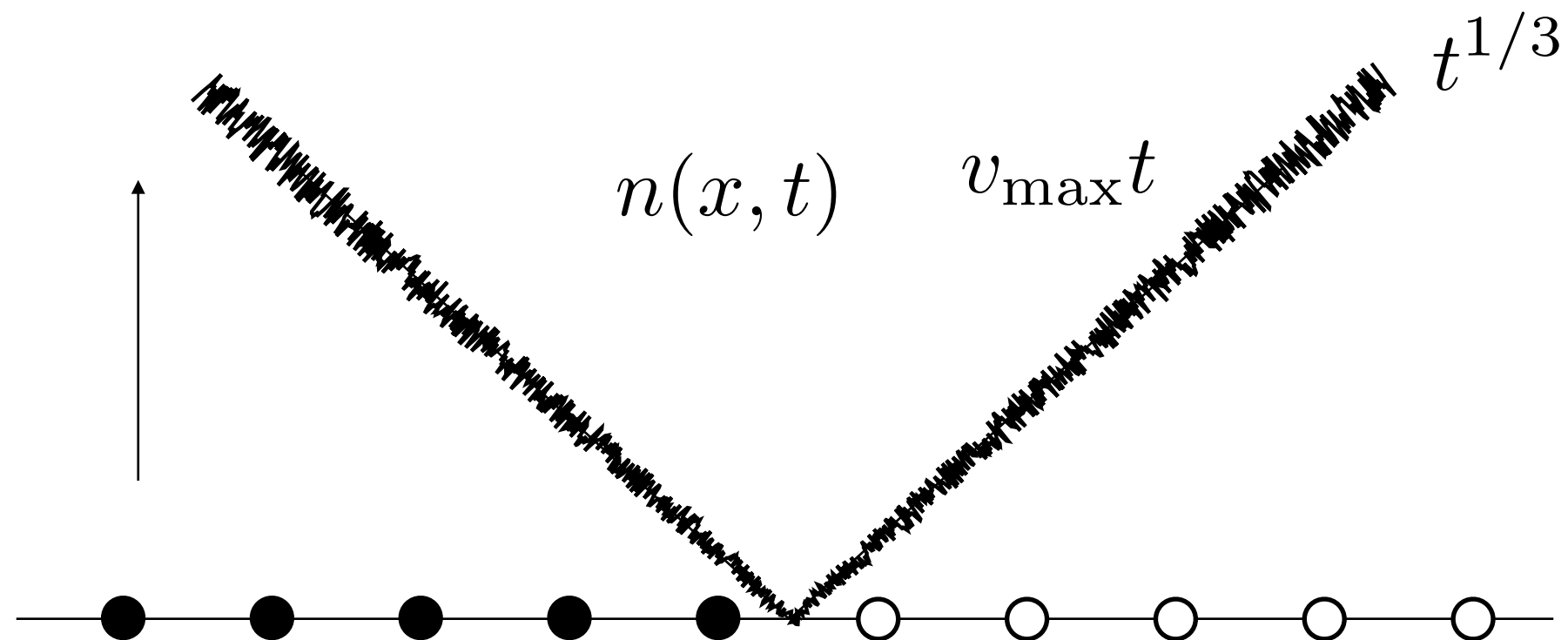
Full Counting Statistics in a Propagating Quantum Front and Random Matrix Spectra

Viktor Eisler

*Vienna Center for Quantum Science and Technology, Faculty of Physics, University of Vienna,
Boltzmanngasse 5, A-1090 Wien, Austria*

Zoltán Rácz

*Institute for Theoretical Physics-HAS, Eötvös University, Pázmány sétány 1/a, 1117 Budapest, Hungary
(Received 20 November 2012; published 5 February 2013)*



KPZ universality class - quantum systems

PRL 117, 070403 (2016)

PHYSICAL REVIEW LETTERS

week ending
12 AUGUST 2016

Exact Short-Time Height Distribution in the One-Dimensional Kardar-Parisi-Zhang Equation and Edge Fermions at High Temperature

Pierre Le Doussal,¹ Satya N. Majumdar,² Alberto Rosso,² and Grégory Schehr²

¹*CNRS-Laboratoire de Physique Théorique de l'Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris Cedex, France*

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(Received 25 April 2016; published 11 August 2016)



KPZ universality class - quantum systems

PHYSICAL REVIEW X 7, 031016 (2017)

Quantum Entanglement Growth under Random Unitary Dynamics

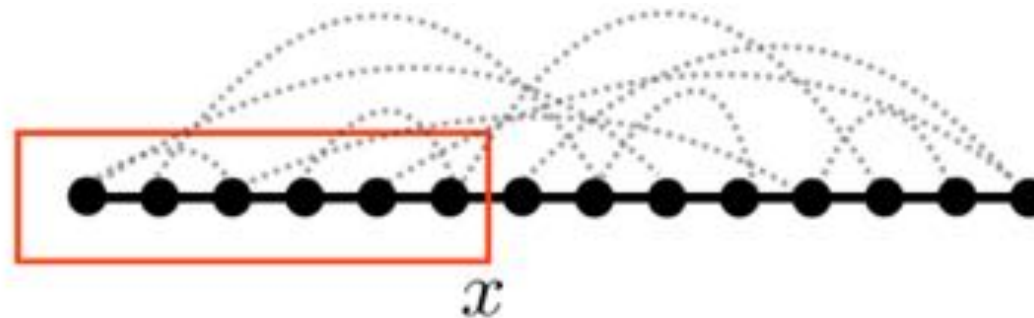
Adam Nahum,^{1,3} Jonathan Ruhman,¹ Sagar Vijay,^{1,2} and Jeongwan Haah¹

¹*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

²*Kavli Institute for Theoretical Physics, University of California Santa Barbara,
Santa Barbara, California 93106, USA*

³*Theoretical Physics, Oxford University, 1 Keble Road, Oxford OX1 3NP, United Kingdom*

(Received 1 September 2016; revised manuscript received 16 April 2017; published 24 July 2017)

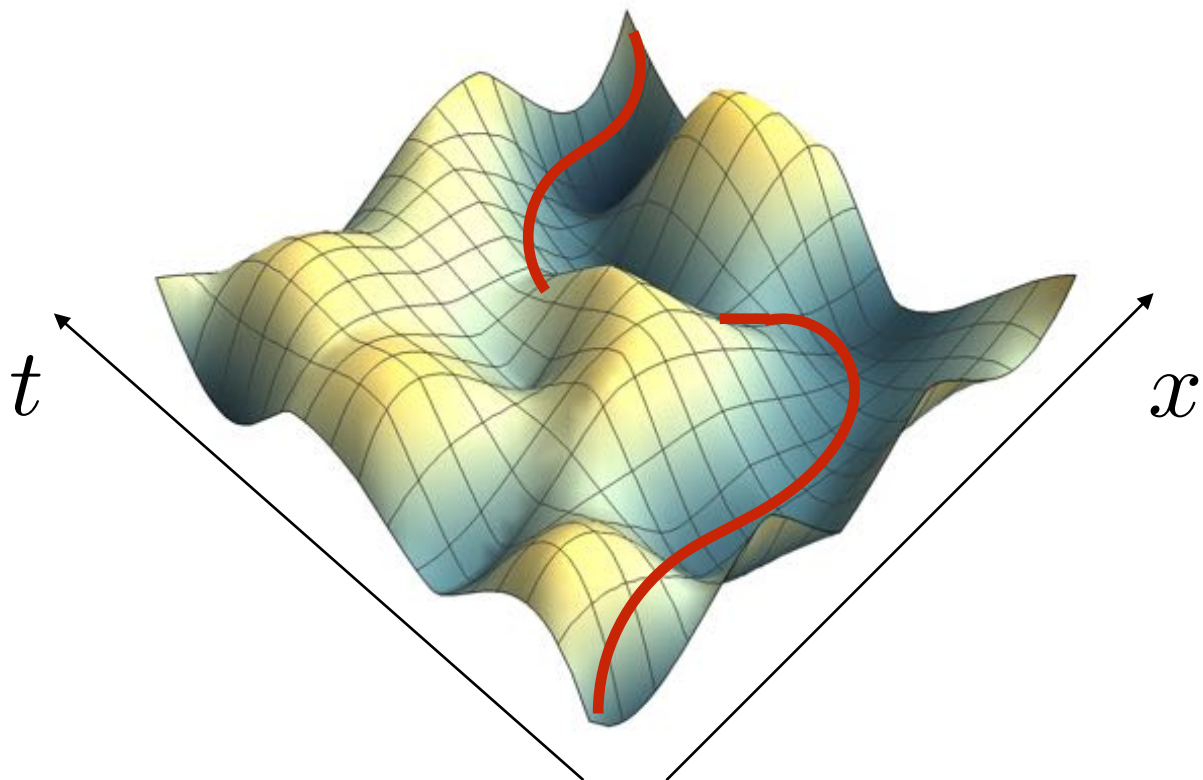


Solution of the KPZ equation via replicas

$$\partial_t h(x, t) = \nu \partial_x^2 h(x, t) + \frac{\lambda_0}{2} (\partial_x h(x, t))^2 + \sqrt{D} \xi(x, t)$$

$$Z(x, t) = e^{h(x, t)}$$

$$\partial_t Z(x, t) = \nu \partial_x^2 Z(x, t) + D \eta(x, t) Z(x, t)$$

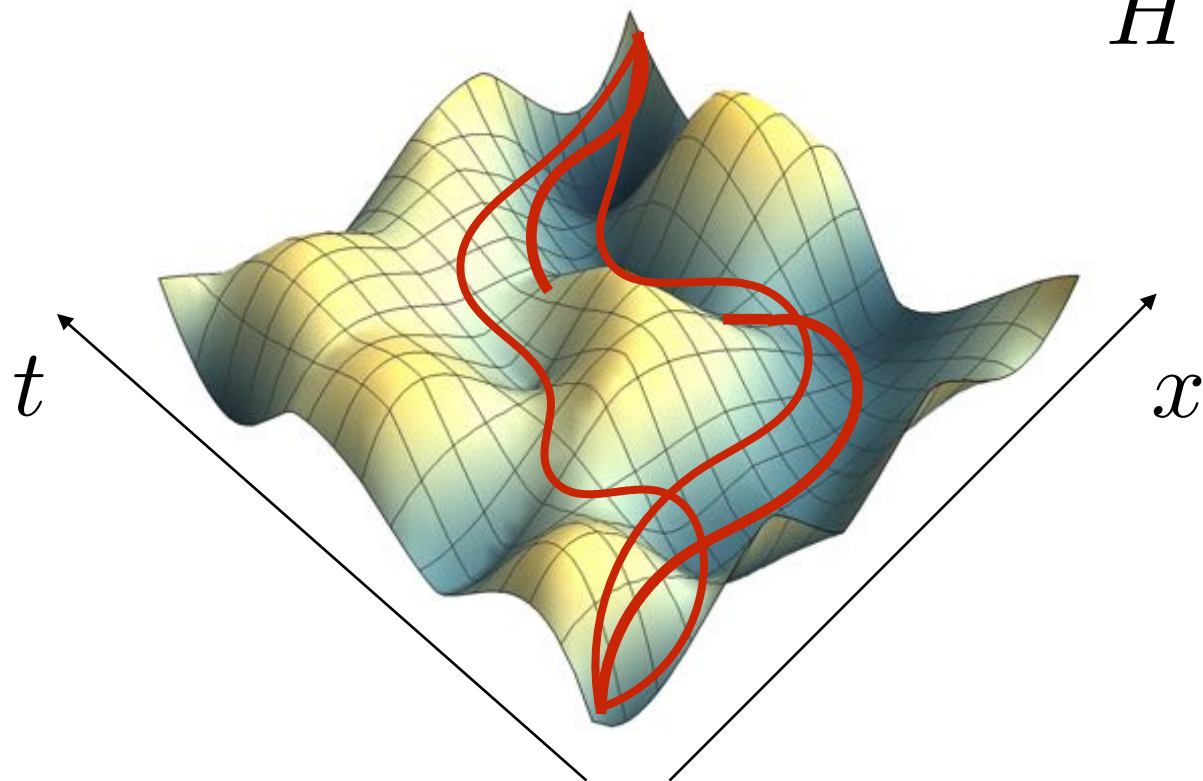


Solution of the KPZ equation via replicas

$$\partial_t Z(x, t) = \nu \partial_x^2 Z(x, t) + D \eta(x, t) Z(x, t)$$

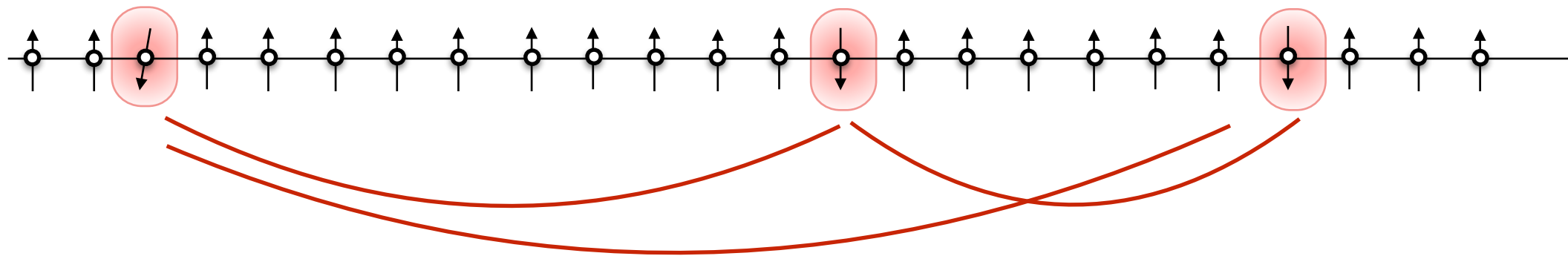
$$\langle e^{nh(x,t)} \rangle = \langle Z^n(x, t) \rangle = \langle \vec{x}_0 | e^{-tH} | \vec{x}, t \rangle$$

$$H = \sum_j \partial_{x_j}^2 - 2\bar{c} \sum_j \delta(x_i - x_j)$$



The attractive Lieb-Liniger model

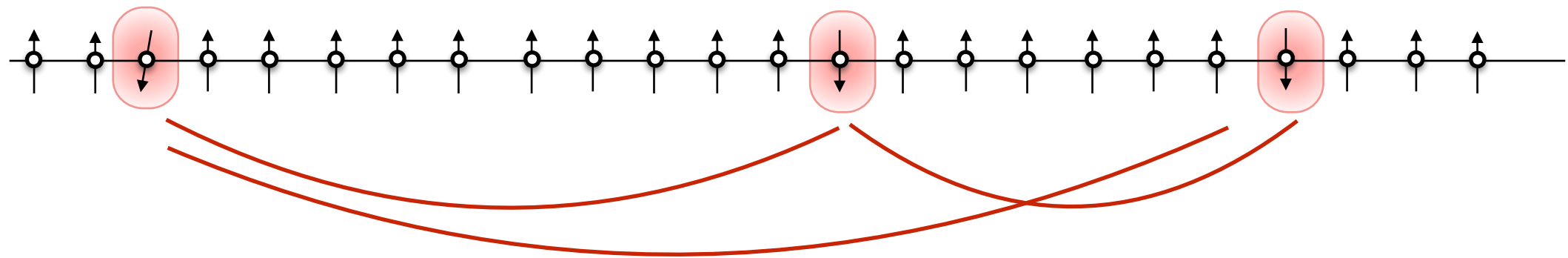
$$H = \sum_j \partial_{x_j}^2 - 2\bar{c} \sum_j \delta(x_i - x_j)$$



$$E(k, s) = sk^2 - \frac{\bar{c}}{12}(s^3 - s)$$

The attractive Lieb-Liniger model

$$H = \sum_j \partial_{x_j}^2 - 2\bar{c} \sum_j \delta(x_i - x_j)$$

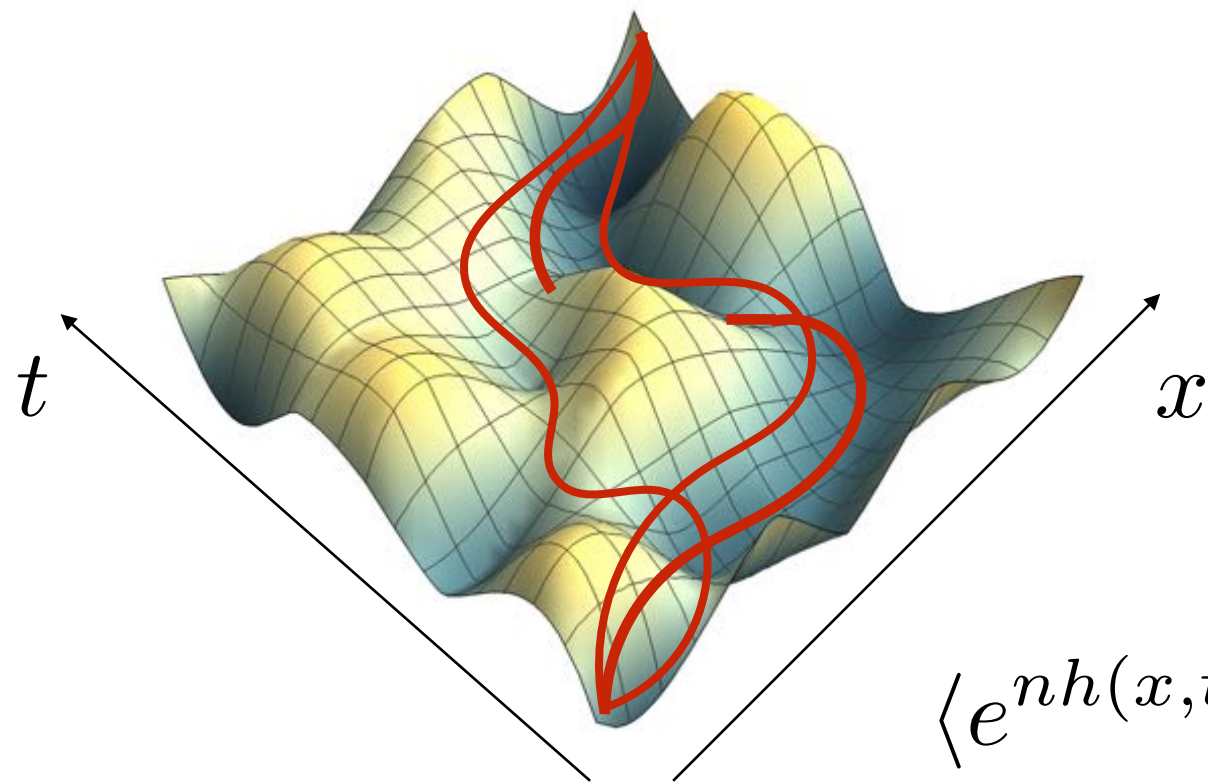


$$E(k, s) = sk^2 - \frac{\bar{c}}{12}(s^3 - s)$$

$$\langle e^{nh(x,t)} \rangle = \langle Z^n(x, t) \rangle = \langle \vec{x}_0 | e^{-tH} | \vec{x}, t \rangle \sim e^{-\bar{c}t/12(n^3 - n)}$$

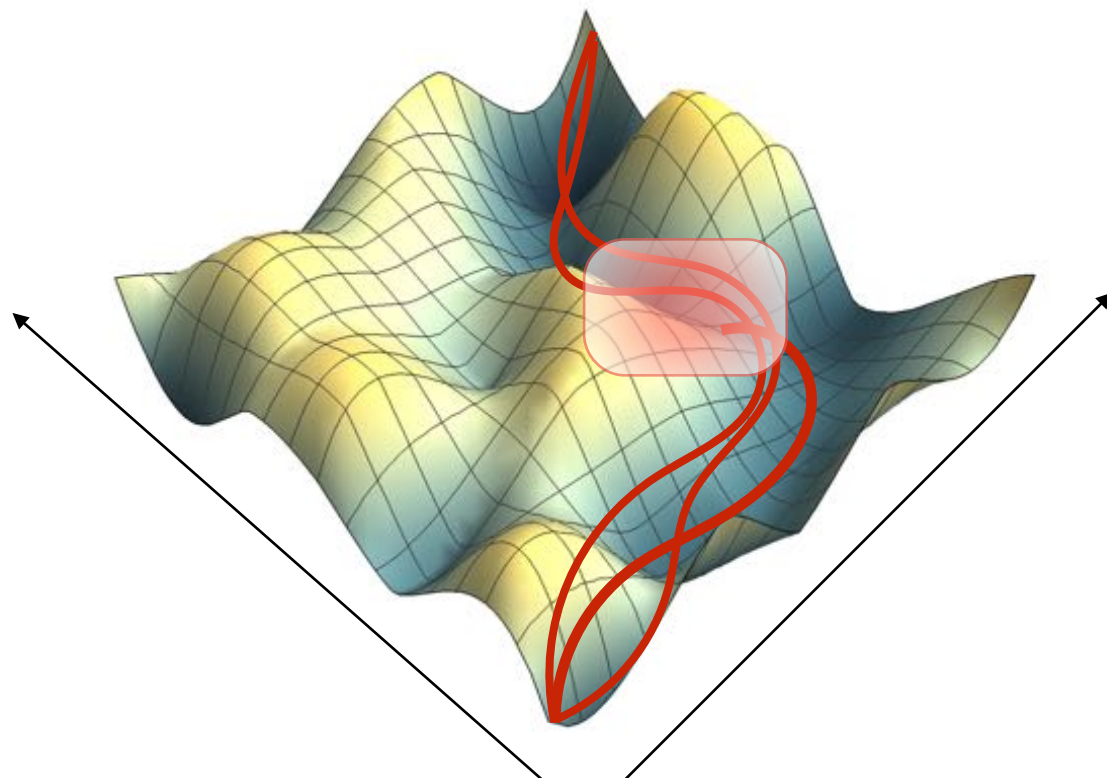
$$n \sim \alpha t^{-1/3} \qquad h(x, t)t^{-1/3} \sim \alpha t^{2/3} + \beta$$

The attractive Lieb-Liniger model



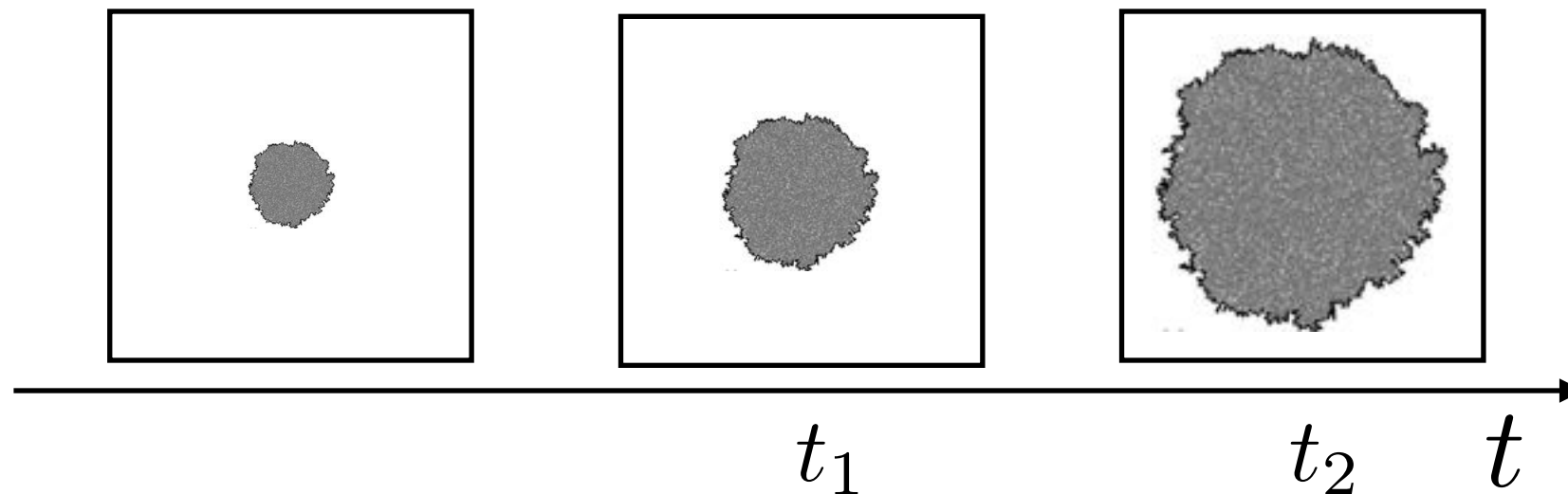
Rare realization

$$\begin{aligned}\langle e^{nh(x,t)} \rangle &= \langle Z^n(x,t) \rangle \\ &= \langle \vec{x}_0 | e^{-tH} | \vec{x}, t \rangle\end{aligned}$$



Typical
realization

Memory in KPZ class

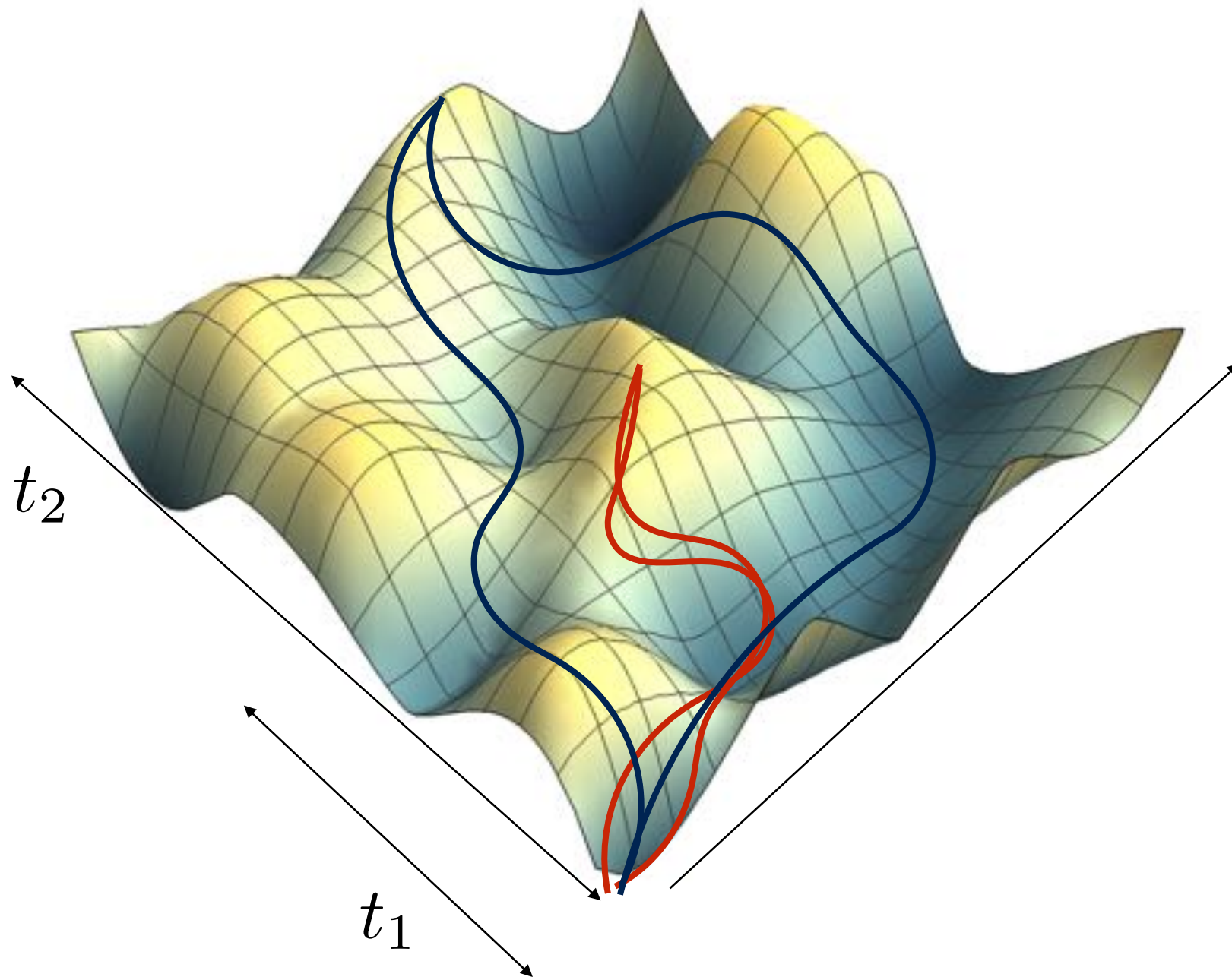


$$\text{cov}(t_1, t_2) = \frac{\langle h(x, t_1) h(x, t_2) \rangle_c}{\langle h(x, t_1)^2 \rangle_c}$$

$$\xrightarrow{t_2/t_1 \rightarrow \infty} 0.623 \dots$$

Memory in the directed polymer

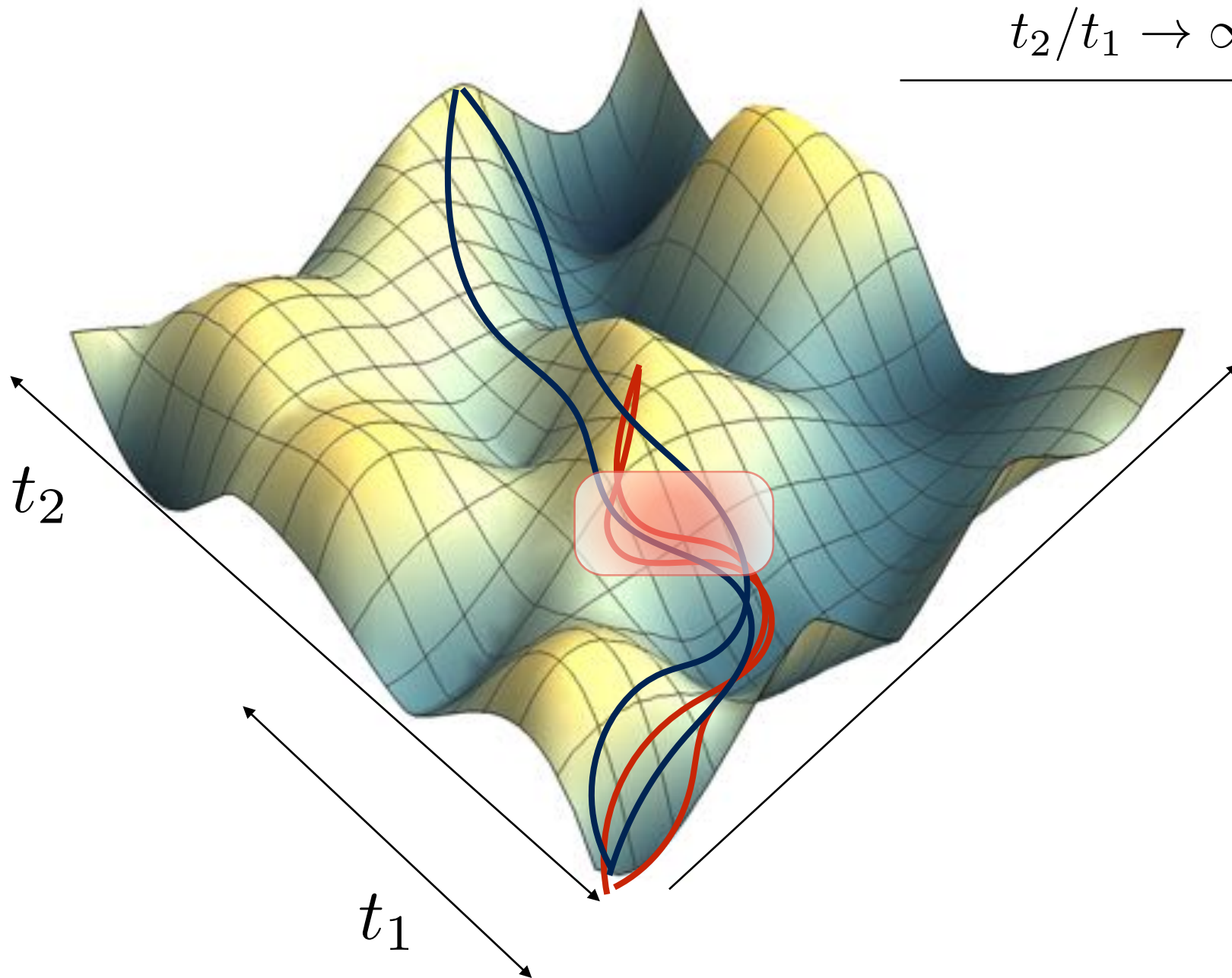
$$\text{cov}(t_1, t_2) = \frac{\langle h(x, t_1) h(x, t_2) \rangle_c}{\langle h(x, t_1)^2 \rangle_c} \propto \text{shared path in } [0, t_1]$$



Memory in the directed polymer

$$\text{cov}(t_1, t_2) = \frac{\langle h(x, t_1) h(x, t_2) \rangle_c}{\langle h(x, t_1)^2 \rangle_c} \propto \text{shared path in } [0, t_1]$$

$\xrightarrow{t_2/t_1 \rightarrow \infty} \propto 0.623 \dots$



Conclusions

Conclusion

GGE steady states in interacting systems

Generalized hydrodynamics and Drude weights

growth phenomena in KPZ equation

Bound states in the steady state

Finite Drude weights in the gapless regime

Divergence of the diffusion constant

Persistence of correlations in the KPZ class

Divergence of the dressed charge of large bound states

$$v_s^{\text{eff}} q_s^{\text{dr}} \sim s$$

Divergence of the binding energy of large bound state

$$E_s \sim s^3$$