Transport and Chaos in Quantum Matter

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Integrable and Chaotic Quantum Dynamics
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Motivation

- A long-standing problem has been to characterize hydrodynamic transport (charge, energy dynamics) in strongly coupled systems.

- Remarkably over the last few years evidence has emerged to suggest that this is related to scrambling and many-body chaos.

- Here I will discuss evidence for two surprising connections between hydrodynamics and the properties of OTOCs.

Part I. Diffusion and the butterfly velocity

MAB -1603.08510 (PRL)
MAB, Davison and Sachdev -1705.07896

Part II. Hydrodynamic modes and chaos

MAB, Lee & Liu - 1801.00010

(related work by Grozdanov, Schalm & Scopelliti - 1710.00921)

Part I

Diffusion and the Butterfly Velocity

Diffusion and Chaos

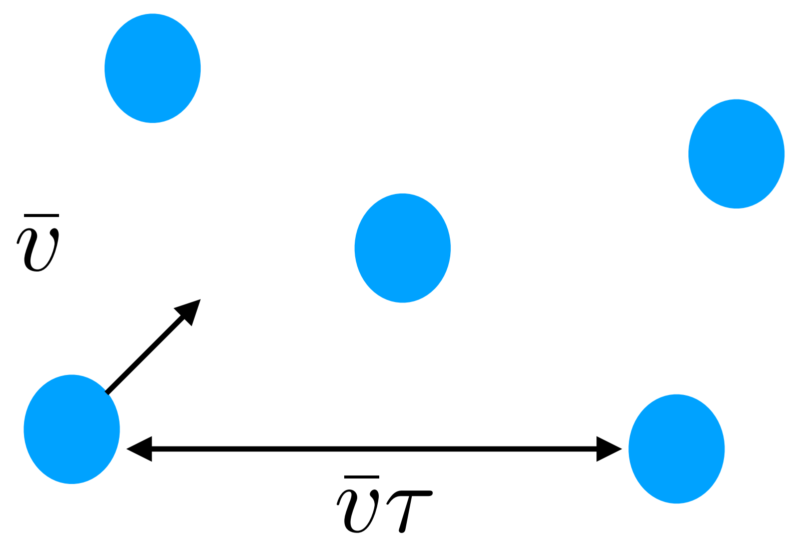
- Important open problem is to understand hydrodynamic transport (conductivity etc) in strongly coupled matter.

- In metals hydrodynamic charge/energy transport is characterized by diffusion

$$\partial_t n = D \nabla^2 n$$

- Conductivities proportional to diffusion constants through Einstein relations.

- In a weakly interacting system



$$D \approx \bar{v}^2 \tau$$

- Transport at strong coupling governed by

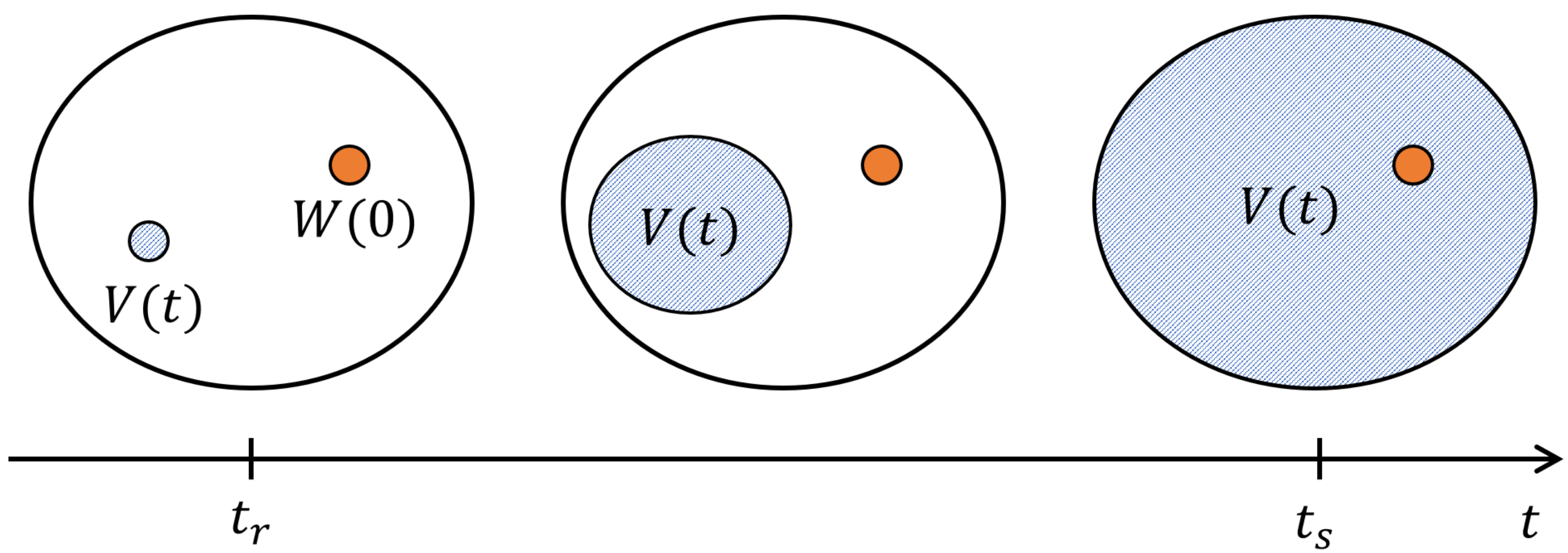
$$\tau \approx \hbar / k_B T$$

Sachdev, Hartnoll

- How can we identify characteristic velocity of diffusion in strongly coupled matter?

Chaos and Scrambling

- Scrambling/chaos describes growth of operators with time



- Simple measure provided by expectation value of commutators

$$C(t) = \langle [V(t), W(0)]^2 \rangle_{\beta_0}$$

- For systems with many degrees of freedom

$$C(t) \sim \frac{1}{\mathcal{N}} e^{\lambda t}$$

$$t_r \ll t \ll t_s$$

$$\lambda \leq \frac{2\pi k_B}{\hbar \beta_0}$$

Maldacena, Shenker & Stanford

- At strong coupling typically get ballistic spreading in space e.g. in SYK chains/holography

$$\langle [V(t,x), W(0,0)]^2 \rangle_{\beta_0} \sim \frac{1}{\mathcal{N}} e^{\lambda(t - x/v_B)}$$

(Different spatial dependence often seen in non-maximally chaotic systems)

Proposal

- Propose v_B as characteristic velocity of diffusion

$$D \approx v_B^2 \tau$$

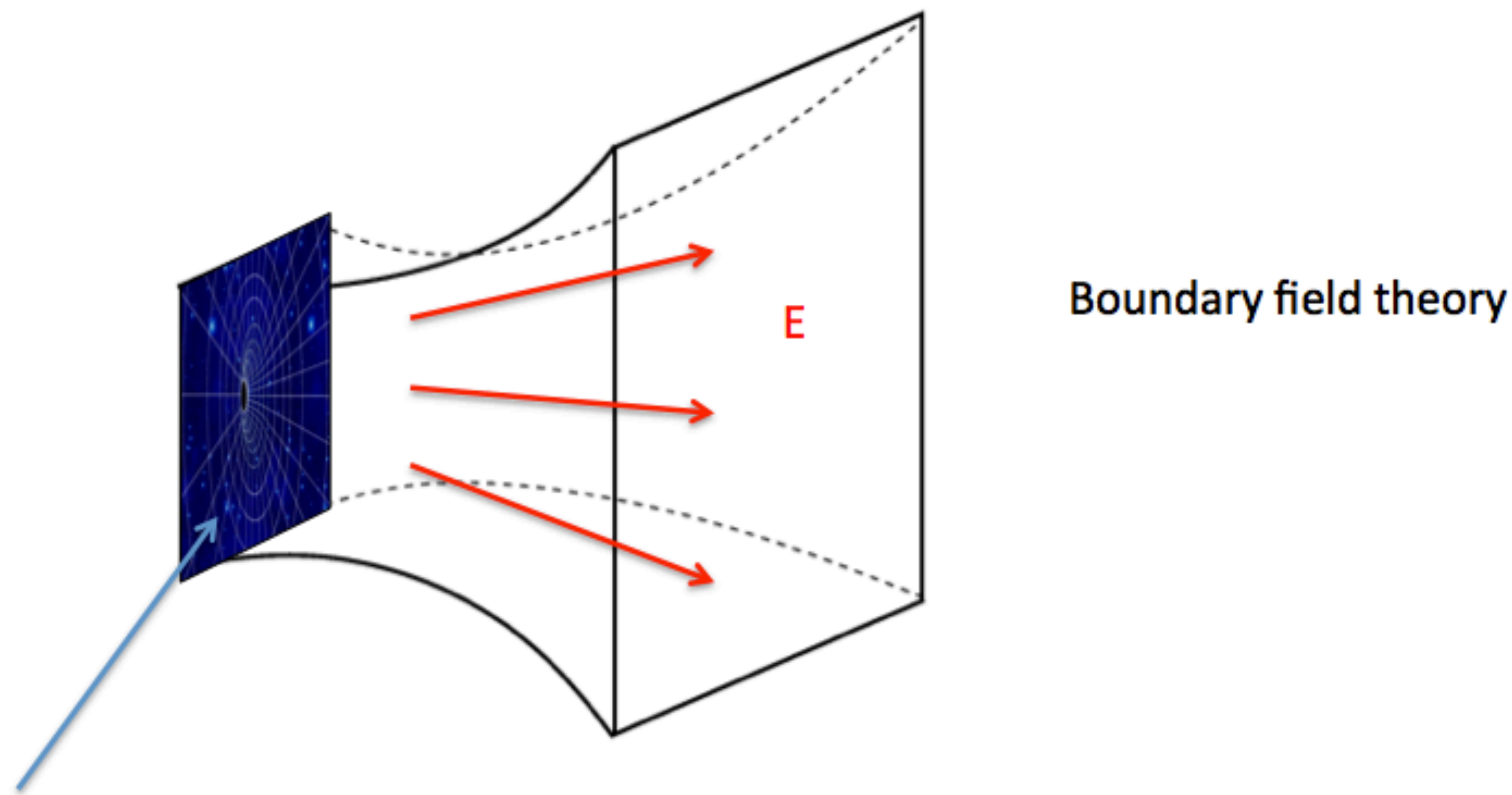
MAB `16 (PRL)

- In many cases also expect Lyapunov exponent to provide measure of mean free time $\tau \approx \tau_L = 1/\lambda$

- Original evidence for this proposal came from studying holographic theories.

Holography

Classical gravity in asymp-AdS spacetime ⟷ Strongly coupled large N gauge theory



- Reissner-Nordstrom black hole
- Hawking radiation = finite temperature, T
- Electric field = chemical potential, μ

- DC thermoelectric conductivities $\sigma \quad \alpha \quad \bar{\kappa}$ can be related to geometry and fields at black hole horizon.

MAB & Tong;
Donos & Gauntlett

- Likewise chaos exponents can be calculated from gravitational shock-wave on horizon.

Shenker & Stanford;
Roberts, Stanford & Susskind

- Diffusion constants proportional to conductivities through Einstein relations.

Holographic examples

- First evidence came from charge diffusion in particle-hole symmetric theories.

$$D_c = \frac{\sigma}{\chi} \qquad \chi = \left(\frac{\partial \rho}{\partial \mu}\right)_T$$

- Calculated this for holographic theories that flow to Lifshitz/hyperscaling geometries in IR.
- Generalised scaling theories described by critical exponents (z, θ, ϕ)

- Shock-wave calculation for these geometries gives

$$\tau_L^{-1} = 2\pi T \qquad v_B^2 \sim L^2 T^{2-2/z}$$

- Found these chaos parameters were universally related to the diffusion constant

$$\boxed{D_c = \frac{d_\theta}{\Delta_\chi} v_B^2 \tau_L}$$

MAB

- Although such a relationship is quite special to particle-hole symmetric theories.

- More general connection is found in relationship between energy/thermal diffusion and chaos

$$D_T = \frac{\kappa}{c_\rho} \qquad c_\rho = T\left(\frac{\partial s}{\partial T}\right)_\rho$$

- For these scaling geometries this is always given by

$$\boxed{D_T = \frac{z}{2z-2} v_B^2 \tau_L}$$

MAB;
MAB, Davison and Sachdev

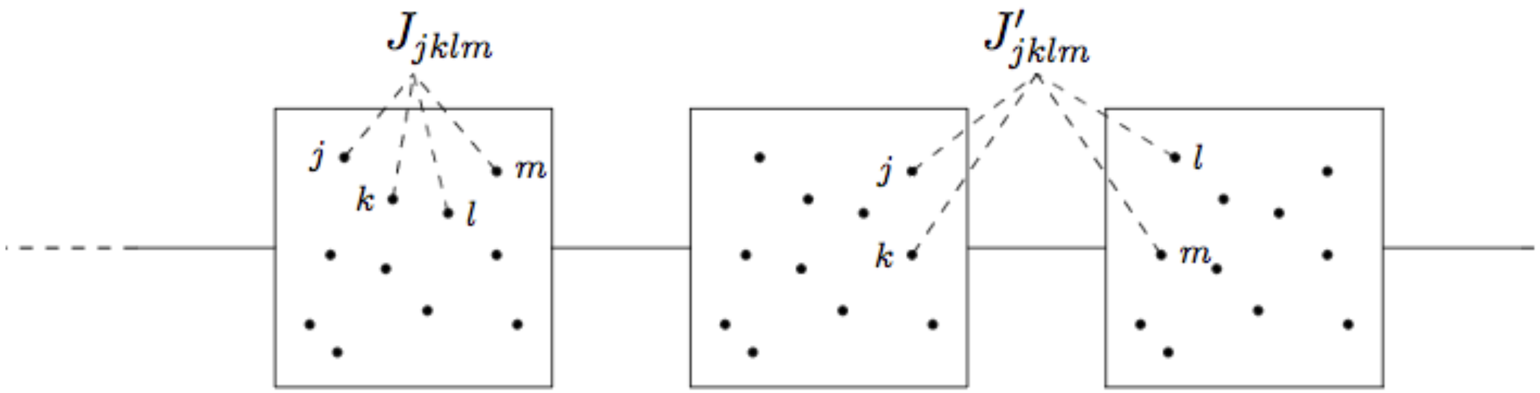
- This holds independently of charge density, magnetic field, periodic potential strength.

Fixed points with z=1 (e.g. CFTs) are a special exception

Other Examples

SYK chains

Gu, Stanford & Qi;
Davison et al



$$\tau_L^{-1} = 2\pi T$$

$$v_B^2 \sim \frac{J'^2 T}{J}$$

$$D_T = v_B^2 \tau_L$$

Critical Fermi surfaces

Quantum chaos on a critical Fermi surface

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str-el] 12 Mar 2017

Abstract

We compute parameters characterizing many-body quantum chaos for a critical Fermi surface without quasiparticle excitations. We examine a theory of N species of fermions at non-zero density coupled to a $U(1)$ gauge field in two spatial dimensions, and determine the Lyapunov rate and the butterfly velocity in an extended random-phase approximation. The thermal diffusivity is found to be universally related to these chaos parameters *i.e.* the relationship is independent of N, the gauge coupling constant, the Fermi velocity, the Fermi surface curvature, and high energy details.

$$\tau_L = \frac{\hbar}{2.48\, k_B T}$$

$$v_B \sim \frac{N v_F^{5/3}}{e^{4/3}\gamma^{1/3}} T^{1/3}$$

$$D_E = 0.42 v_B^2 \tau_L$$

Patel & Sachdev

- Connections of the form $D \sim v_B^2 \tau_L$ have also been seen in

Electron/phonon bad metals - Werman, Kivelson & Berg
Incoherent Bose-Hubbard models - Bohrdt, Endrel, Mendes & Knap
O(N) models - Chowdury & Swingle
Fermi liquids with disorder/electron-electron interactions - Aleiner, Faoro & Ioffe
Higher derivative holographic theories - Baggioli, Gouteraux, Kiritsis & Li

- Hints towards a general relationship between energy dynamics and chaos?

Part II

Hydrodynamic modes and chaos

Hydrodynamics and chaos

- These results constitute part of growing evidence for a more fundamental connection between chaos and energy dynamics.

e.g. holography (gravity), SYK (Schwarzian)…

- Recently we proposed an effective description of chaos where energy dynamics and exponential growth of OTOCs are governed by the same hydrodynamic mode $\sigma(t)$

MAB, Lee & Liu

- This theory predicts a precise signature of chaos in the energy density two point function.

- At low frequencies the retarded energy density two point function typically has a hydrodynamic diffusion pole *

$$\omega(k) = -iD_T k^2 + \dots$$

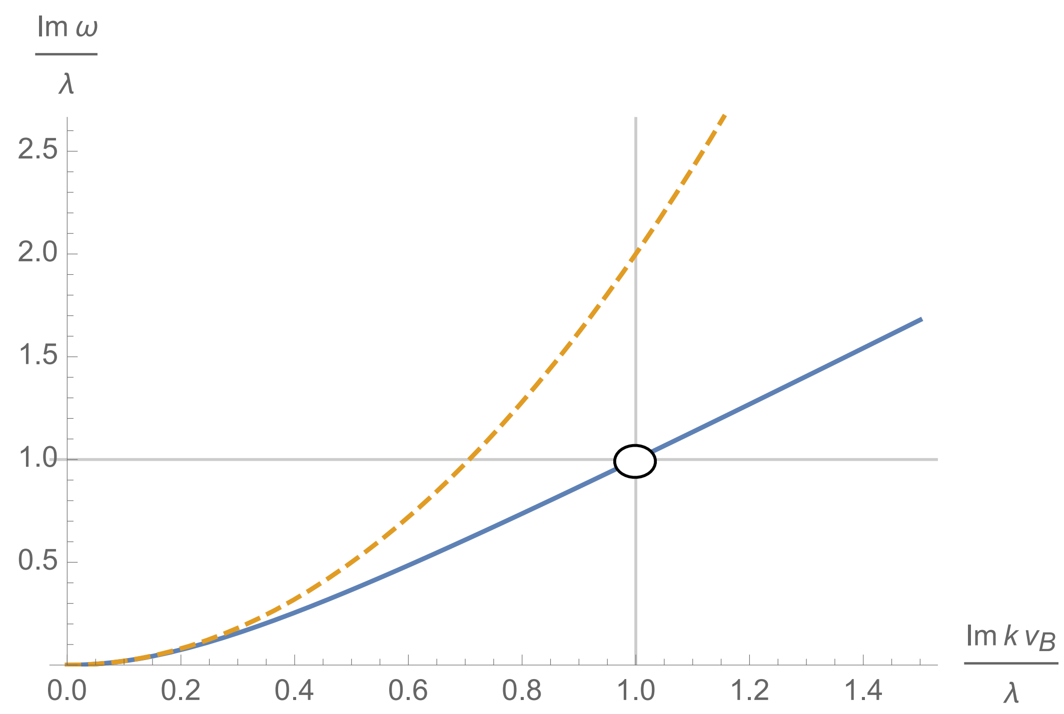
- Chaos is related to the dispersion relation for this pole at energy scales $\omega \approx i\lambda$

* for systems with momentum dissipation

- Prediction 1: This pole must pass through point

$$\omega = i\lambda \quad k = \frac{i\lambda}{v_B}$$

- Prediction 2: At this point the residue should vanish (`Pole-skipping')



- Remarkably precisely these features had previously been found to hold for the hydrodynamic (sound) mode of a holographic theory dual to an AdS5-Schwarzchild black hole.

Grozdanov, Schalm & Scopelliti (PRL)

- They can also be seen to be true for SYK chains.

Gu, Qi & Stanford

Conclusions

- Discussed evidence for two surprising connections between hydrodynamic transport and chaos.
- First identified using holographic theories but have since been seen more generally.
- Hints at a fundamental connection between these phenomena?

See Hong's talk on Wednesday

Thank you!